Linear quantum regfrigerators: the ultimate limit for cooling and the origin of the third law

Nahuel Freitas, Juan Pablo Paz

Departamento de Fsica - Facultad de Ciencias Exactas y Naturales - Universidad de Buenos Aires

6 de Junio de 2015

Outline

- Motivation and context. Third Law of Thermodynamics (Unattainability principle)
- A general theory of linear quantum refrigerators
- Pairs creation as a fundamental limit for cooling (dynamical Casimir effect)
- Failure of the weak coupling approximation
- Minimum achievable temperature
- Conclusions and next steps

Motivation and context

Quantum thermodynamics studies thermodynamical processes in quantum systems from first principles, without assuming any of the usual thermodynamical laws. It allows to:

Motivation and context

Quantum thermodynamics studies thermodynamical processes in quantum systems from first principles, without assuming any of the usual thermodynamical laws. It allows to:

- Understand the emergence of thermodynamics from quantum dynamics

Motivation and context

Quantum thermodynamics studies thermodynamical processes in quantum systems from first principles, without assuming any of the usual thermodynamical laws. It allows to:

- Understand the emergence of thermodynamics from quantum dynamics
- Describe processes in systems with few degrees of freedom

Motivation and context

Quantum thermodynamics studies thermodynamical processes in quantum systems from first principles, without assuming any of the usual thermodynamical laws. It allows to:

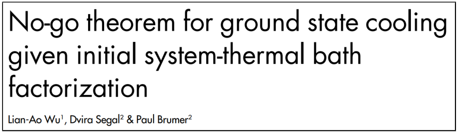
- Understand the emergence of thermodynamics from quantum dynamics
- Describe processes in systems with few degrees of freedom
- Describe processes arbitrarily away from thermal equillibrium

The dynamical third law

The 1^{st} and 2^{nd} laws can and have been derived from first principles in a variety of settings. Some controversies exists regarding the dynamical third law.

Nernst statement

'It is impossible for any process, no matter how idealized, to reduce the entropy of a system to its absolute-zero value in a finite number of operations'

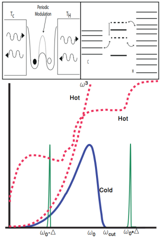


No-go theorem for ground state cooling given initial system-thermal bath factorization

Lian-Ao Wu[1], Dvira Segal[2] & Paul Brumer[2]

Quantum Bath Refrigeration towards Absolute Zero: Challenging the Unattainability Principle

M. Kolář, D. Gelbwaser-Klimovsky, R. Alicki, and G. Kurizki
Phys. Rev. Lett. **109**, 090601 – Published 27 August 2012

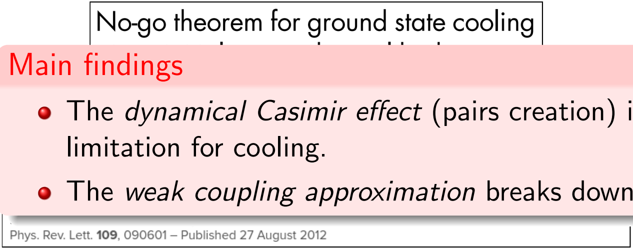


The dynamical third law

The 1^{st} and 2^{nd} laws can and have been derived from first principles in a variety of settings. Some controversies exists regarding the dynamical third law.

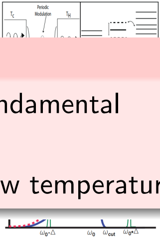
Nernst statement

'It is impossible for any process, no matter how idealized, to reduce the entropy of a system to its absolute-zero value in a finite number of operations'



No-go theorem for ground state cooling

Phys. Rev. Lett. **109**, 090601 – Published 27 August 2012



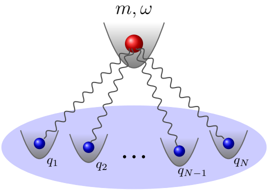
Main findings

- The *dynamical Casimir effect* (pairs creation) is a fundamental limitation for cooling.
- The *weak coupling approximation* breaks down for low temperatures.

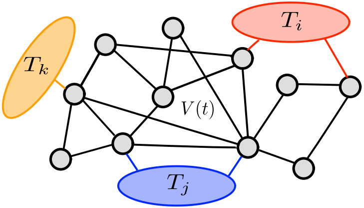
A general theory of linear quantum refrigerators

Model

Quantum Brownian Motion:



Generalized QBM:

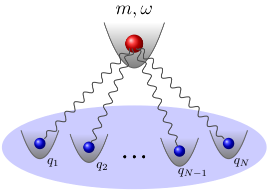


$$H_S(t) = \frac{1}{2}P^T M^{-1} P + \frac{1}{2} X^T V(t) X$$

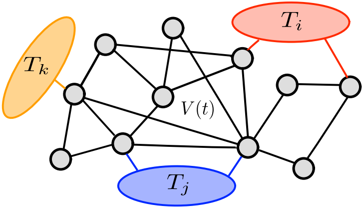
A general theory of linear quantum refrigerators

Model

Quantum Brownian Motion:



Generalized QBM:



$$H_S(t) = \frac{1}{2} P^T M^{-1} P + \frac{1}{2} X^T V(t) X$$

A periodic parametric driving is performed on the network:

$$V(t) = V(t + 2\pi/\omega_d) = \sum_{k=-\infty}^{k=+\infty} V_k e^{i\omega_d t}$$

What is the asymptotic state of the network? What are the heat currents? How can we cool a given reservoir?

A general theory of linear quantum refrigerators

Methods

- For bosonic reservoirs the problem can be solved exactly

$$H_T = H_S(t) + \sum_\alpha H_{E_\alpha} + \sum_\alpha H_{int,\alpha}$$

$$H_{E,\alpha} = \sum_j (\pi^2_{\alpha,j}/2m + m\omega^2_{\alpha,j} q^2_{\alpha,j}/2) \qquad H_{int,\alpha} = \sum_{j,k} C_{\alpha,jk}\; X_j\; q_{\alpha,k}$$

A general theory of linear quantum refrigerators

Methods

- For bosonic reservoirs the problem can be solved exactly

$$H_T = H_S(t) + \sum_\alpha H_{E_\alpha} + \sum_\alpha H_{int,\alpha}$$

$$H_{E,\alpha} = \sum_j (\pi^2_{\alpha,j}/2m + m\omega^2_{\alpha,j} q^2_{\alpha,j}/2) \qquad H_{int,\alpha} = \sum_{j,k} C_{\alpha,jk}\ X_j\ q_{\alpha,k}$$

- The central ingredient is the Green's function of the system.

$$\ddot{g}(t,t') + V_R(t) g(t,t') + \int_0^t d\tau\ \gamma(t-\tau)\dot{g}(\tau,t') = \delta(t-t')\mathbb{1}$$

$$\gamma(t) = \int_0^\infty d\omega\ I(\omega)\cos(\omega t)/\omega \qquad [I_\alpha(\omega)]_{j,k} = \sum_l \frac{1}{m\omega} C_{\alpha,jl} C_{\alpha,kl} \delta(\omega - \omega_{\alpha,l})$$

A general theory of linear quantum refrigerators

Methods

We assume:

- Stable dynamics
- Long times
- Periodic driving

$$p(\omega,t)=\int_0^t g(t,t')e^{i\omega(t'-t)}dt'=\sum_k A_k(\omega)e^{ik\omega_d t}$$

$$\hat{g}\left(i(\omega+k\omega_d)\right)^{-1}A_k(\omega)+\sum_{j\neq k}V_jA_{k-j}(\omega)=\mathbb{1}\delta_{k,0}.$$

Under those conditions the asymptotic state is also periodic. For example:

$$\sigma^{xp}(t)=\frac{1}{2}\langle XP^T+PX^T\rangle(t)=\mathrm{Im}\left[\sum_{j,k}\sigma^{xp}_{j,k}\ e^{i\omega_d(j-k)t}\right]$$

where:

$$\sigma^{xp}_{j,k}=\frac{\hbar}{2}\int_0^\infty d\omega\ (\omega+k\omega_d)\ A_j(\omega)\tilde{\nu}(\omega)A_k^\dagger(\omega)$$

A general theory of linear quantum refrigerators

Definitions of work and heat rates

$$\frac{d}{dt}\langle H_S\rangle(t) = \sum_\alpha \underbrace{\frac{1}{i\hbar}\langle[H_S, H_{int,\alpha}]\rangle}_{\dot{Q}_\alpha} + \underbrace{\frac{\partial}{\partial t}\langle H_S\rangle}_{\dot{W}}$$

We obtain the instantaneous rates:

$$\dot{W}(t) = \frac{1}{2}\,\mathrm{Tr}\left[\dot{V}(t)\sigma^{xx}(t)\right] \qquad \dot{Q}_\alpha = \frac{1}{2}\,\mathrm{Tr}\left[P_\alpha\dot{\sigma}^{pp}(t)M^{-1}\right] + \mathrm{Tr}\left[P_\alpha V(t)\sigma^{xp}(t)M^{-1}\right]$$

In the asymptotic state, we define the average rates per cycle:

$$\dot{\bar{W}} = \frac{1}{\tau}\lim_{n\to\infty}\int_{n\,\tau}^{(n+1)\tau}\dot{W}(t)\;dt \qquad \dot{\bar{Q}}_\alpha = \frac{1}{\tau}\lim_{n\to\infty}\int_{n\tau}^{(n+1)\tau}\dot{Q}(t)\;dt$$

Which satisfy:

$$\boxed{\dot{\bar{W}} + \sum_\alpha \dot{\bar{Q}}_\alpha = 0} \qquad \boxed{\sum_\alpha \frac{-\dot{\bar{Q}}_\alpha}{T_\alpha} \geq 0}$$

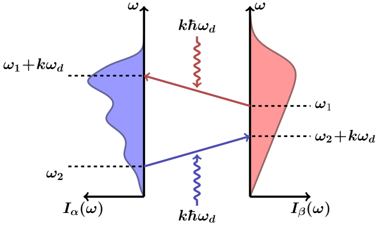
Final result for the heat rates

$$\boxed{\dot{Q}_\alpha \;=\; \dot{Q}_\alpha^{\mathsf{RP}} + \dot{Q}_\alpha^{\mathsf{RH}} + \dot{Q}_\alpha^{\mathsf{NRH}}}$$

Final result for the heat rates

$$\boxed{\dot{Q}_\alpha = \underbrace{\dot{Q}_\alpha^{\text{RP}} + \dot{Q}_\alpha^{\text{RH}}}_{(1)} + \dot{Q}_\alpha^{\text{NRH}}}$$

(1) Resonant processes



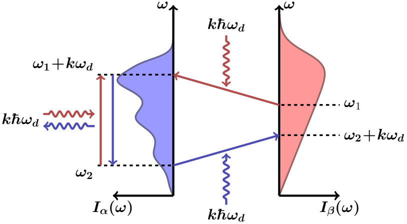
$$\dot{Q}_\alpha^{\text{RP}} = \sum_{\beta\neq\alpha}\sum_k \int_{0'}^{\infty} d\omega\ \hbar\omega\ p^k_{\beta,\alpha}(\omega)\ N_\alpha(\omega) - $$
$$- \sum_{\beta\neq\alpha}\sum_k \int_{0'}^{\infty} d\omega\ \hbar(\omega + k\omega_d)\ p^k_{\alpha,\beta}(\omega)\ N_\beta(\omega)$$

$$p^{(k)}_{\alpha,\beta}(\omega) = \frac{\pi}{2}\,\mathrm{Tr}[I_\alpha(|\omega + k\omega_d|)A_k(\omega)I_\beta(\omega)A_k^\dagger(\omega)]$$

Final result for the heat rates

$$\boxed{\dot{Q}_\alpha \;=\; \underbrace{\dot{Q}_\alpha^{\mathsf{RP}} + \dot{Q}_\alpha^{\mathsf{RH}}}_{(1)} + \dot{Q}_\alpha^{\mathsf{NRH}}}$$

(1) Resonant processes



$$\dot{Q}_\alpha^{\mathsf{RP}} = \sum_{\beta\neq\alpha}\sum_k \int_{0'}^{\infty} d\omega\ \hbar\omega\ p^k_{\beta,\alpha}(\omega)\ N_\alpha(\omega) -$$

$$-\sum_{\beta\neq\alpha}\sum_k \int_{0'}^{\infty} d\omega\ \hbar(\omega + k\omega_d)\ p^k_{\alpha,\beta}(\omega)\ N_\beta(\omega)$$

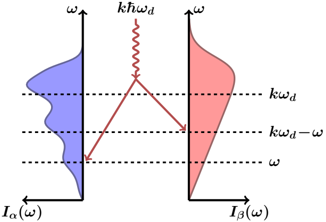
$$\dot{Q}_\alpha^{\mathsf{RH}} = -\sum_{\beta\neq\alpha}\sum_k \int_{0'}^{\infty} d\omega\ k\hbar\omega_d\ p^k_{\alpha,\alpha}(\omega)\ N_\alpha(\omega)$$

$$p^{(k)}_{\alpha,\beta}(\omega) = \frac{\pi}{2}\,\mathrm{Tr}[I_\alpha(|\omega + k\omega_d|)A_k(\omega)I_\beta(\omega)A_k^\dagger(\omega)]$$

Final result for the heat rates

$$\boxed{\dot{Q}_\alpha = \dot{Q}_\alpha^{\mathsf{RP}} + \dot{Q}_\alpha^{\mathsf{RH}} + \underbrace{\dot{Q}_\alpha^{\mathsf{NRH}}}_{(2)}}$$

(2) Non-resonant heating



$$\dot{Q}_\alpha^{\mathsf{NRH}} = -\sum_{k>0}\int_0^{k\omega_d} d\omega\; k\hbar\omega_d\; p_{\alpha,\alpha}^{-k}(\omega)\;\left(N_\alpha(\omega)+\frac{1}{2}\right) -$$

$$-\sum_{\beta\neq\alpha}\sum_{k>0}\int_0^{k\omega_d} d\omega\; \hbar(k\omega_d-\omega)\; p_{\alpha,\beta}^{-k}(\omega)\;\left(N_\beta(\omega)+\frac{1}{2}\right) -$$

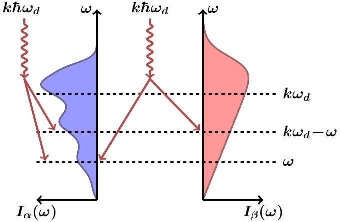
$$-\sum_{\beta\neq\alpha}\sum_{k>0}\int_0^{k\omega_d} d\omega\; \hbar\omega\; p_{\beta,\alpha}^{-k}(\omega)\;\left(N_\alpha(\omega)+\frac{1}{2}\right)$$

$$p_{\alpha,\beta}^{(k)}(\omega) = \frac{\pi}{2}\,\mathrm{Tr}[I_\alpha(|\omega+k\omega_d|)A_k(\omega)I_\beta(\omega)A_k^\dagger(\omega)]$$

Final result for the heat rates

$$\boxed{\dot{Q}_\alpha = \dot{Q}_\alpha^{\text{RP}} + \dot{Q}_\alpha^{\text{RH}} + \underbrace{\dot{Q}_\alpha^{\text{NRH}}}_{(2)}}$$

(2) Non-resonant heating



$$\dot{Q}_\alpha^{\text{NRH}} = -\sum_{k>0}\int_0^{k\omega_d} d\omega\; k\hbar\omega_d\; p_{\alpha,\alpha}^{-k}(\omega)\;\left(N_\alpha(\omega)+\frac{1}{2}\right) -$$

$$-\sum_{\beta\neq\alpha}\sum_{k>0}\int_0^{k\omega_d} d\omega\; \hbar(k\omega_d-\omega)\; p_{\alpha,\beta}^{-k}(\omega)\;\left(N_\beta(\omega)+\frac{1}{2}\right) -$$

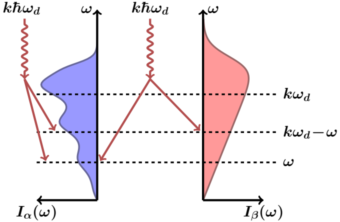
$$-\sum_{\beta\neq\alpha}\sum_{k>0}\int_0^{k\omega_d} d\omega\; \hbar\omega\; p_{\beta,\alpha}^{-k}(\omega)\;\left(N_\alpha(\omega)+\frac{1}{2}\right)$$

$$p_{\alpha,\beta}^{(k)}(\omega) = \frac{\pi}{2}\,\mathrm{Tr}[I_\alpha(|\omega+k\omega_d|)A_k(\omega)I_\beta(\omega)A_k^\dagger(\omega)]$$

Final result for the heat rates

$$\boxed{\dot{Q}_\alpha = \dot{Q}^{\text{RP}}_\alpha + \dot{Q}^{\text{RH}}_\alpha + \underbrace{\dot{Q}^{\text{NRH}}_\alpha}_{(2)}}$$

(2) Non-resonant heating



$$\dot{Q}^{\text{NRH}}_\alpha = -\sum_{k>0}\int_0^{k\omega_d} d\omega\ k\hbar\omega_d\ p^{-k}_{\alpha,\alpha}(\omega)\left(N_\alpha(\omega)+\frac{1}{2}\right) - \\ - \sum_{\beta\neq\alpha}\sum_{k>0}\int_0^{k\omega_d} d\omega\ \hbar(k\omega_d-\omega)\ p^{-k}_{\alpha,\beta}(\omega)\left(N_\beta(\omega)+\frac{1}{2}\right) - \\ - \sum_{\beta\neq\alpha}\sum_{k>0}\int_0^{k\omega_d} d\omega\ \hbar\omega\ p^{-k}_{\beta,\alpha}(\omega)\left(N_\alpha(\omega)+\frac{1}{2}\right)$$

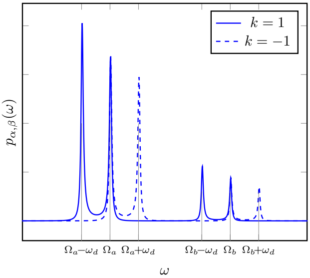
The unattainability principle holds!

At zero temperature only $\dot{Q}^{\text{NRH}}_\alpha < 0$ survives!!
There is always a minimum temperature that supports cooling

Scaling laws

$$\dot{Q}^{\mathrm{RP}}_{\alpha\to\beta} = \sum_k \int_{0'}^{\infty} d\omega\ \hbar\omega\ p^{k}_{\beta,\alpha}(\omega)\ N_\alpha(\omega)$$

$$\dot{Q}^{\mathrm{NRH}}_{\alpha} = -\sum_{k>0} \int_{0}^{k\omega_d} d\omega\ k\hbar\omega_d\ p^{-k}_{\alpha,\alpha}(\omega)\ \left(N_\alpha(\omega) + \frac{1}{2}\right)$$



$$p^{(k)}_{\alpha,\beta}(\omega) = \frac{\pi}{2}\,\mathrm{Tr}[I_\alpha(|\omega + k\omega_d|)A_k(\omega)I_\beta(\omega)A^\dagger_k(\omega)]$$

If $I_\alpha(\omega) \propto \gamma_0$, then:

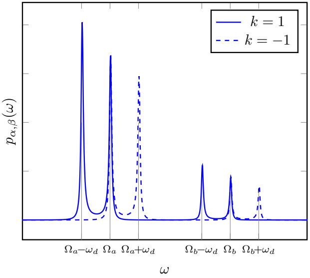
$$\dot{Q}^{\mathrm{RP}}_{\alpha} \propto \gamma_0$$
$$\dot{Q}^{\mathrm{NRH}}_{\alpha} \propto \gamma_0^2$$

Scaling laws

$$\dot{Q}^{\mathrm{RP}}_{\alpha\to\beta} = \sum_k \int_{0'}^{\infty} d\omega\ \hbar\omega\ p^{k}_{\beta,\alpha}(\omega)\ N_\alpha(\omega)$$

$$\dot{Q}^{\mathrm{NRH}}_{\alpha} = -\sum_{k>0} \int_{0}^{k\omega_d} d\omega\ k\hbar\omega_d\ p^{-k}_{\alpha,\alpha}(\omega)\ \left(N_\alpha(\omega)+\frac{1}{2}\right)$$

$$p^{(k)}_{\alpha,\beta}(\omega) = \frac{\pi}{2}\,\mathrm{Tr}[I_\alpha(|\omega + k\omega_d|)A_k(\omega)I_\beta(\omega)A^{\dagger}_{k}(\omega)]$$



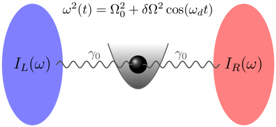
If $I_\alpha(\omega) \propto \gamma_0$, then:

$$\dot{Q}^{\mathrm{RP}}_{\alpha} \propto \gamma_0$$
$$\dot{Q}^{\mathrm{NRH}}_{\alpha} \propto \gamma_0^2$$

Failure of the weak coupling approximation

Pairs creation into the reservoirs is a next-to-leading order effect in the weak coupling aproximation (fourth order in the interaction Hamiltonian)

Minimum temperature for a simple case



$$\dot{Q}_L^{\text{RP}} = e^{-\frac{\Omega_0 - \omega_d}{T_0}} \frac{|\delta\Omega|^4(\pi^2/8)}{\Omega_0^2(\Omega_0^2 - (\Omega_0 - \omega_d)^2)^2}$$
$$\times \frac{1}{\Gamma_0}\left\{I_R^0(\Omega_0) I_L^0(\Omega_0 - \omega_d)(\Omega_0 - \omega_d) - \Omega_0 I_L^0(\Omega_0) I_R^0(\Omega_0 - \omega_d)\right\}$$

- If $I_L(\omega) = I_R(\omega)$, then $Q_L^{\text{RP}} \leq 0$. No symmetry broken.
- If $I_L(\Omega_0) \ll I_R(\Omega_0)$, then $Q_L^{\text{RP}} \geq 0$. Heat extracted from L.

Adaptative strategy - Challenging the third law

$$\frac{dT_L}{dt} = -\frac{1}{C_v}\dot{Q}_L$$

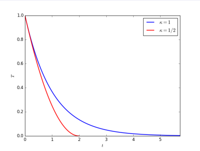
If we choose adjust ω_d as $\omega_d \simeq \Omega_0 - T_L$:

$$\dot{Q}_L^{\text{RP}} = \frac{\pi^2}{8e}\frac{\delta\Omega^4}{\Omega_0^6} T_L I_L(T_L) \frac{I_R^0(\Omega_0)}{\Gamma_0} \propto \gamma_0 T_L^{1+\lambda_L}$$

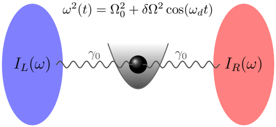
for $I_L(\omega) \propto \gamma_0\, \omega^{\lambda_L}$

Tipically, $C_v \propto T_L^d$, and therefore:

$$\frac{dT_L}{dt} = -\frac{1}{C_v}\dot{Q}_L \propto -\gamma_0 \eta T_L^{\overbrace{1 + \lambda_L - d}^{\kappa}}$$



Minimum temperature for a simple case



$$\dot{Q}_L^{\mathrm{RP}} = e^{-\frac{\Omega_0 - \omega_d}{T_0}} \frac{|\delta\Omega|^4(\pi^2/8)}{\Omega_0^2(\Omega_0^2 - (\Omega_0 - \omega_d)^2)^2}$$
$$\times \frac{1}{\Gamma_0}\left\{ I_R^0(\Omega_0) I_L^0(\Omega_0 - \omega_d)(\Omega_0 - \omega_d) - \cancel{\Omega_0 I_L^0(\Omega_0) I_R^0(\Omega_0 - \omega_d)} \right\}$$

- If $I_L(\omega) = I_R(\omega)$, then $Q_L^{\mathrm{RP}} \leq 0$. No symmetry broken.
- If $I_L(\Omega_0) \ll I_R(\Omega_0)$, then $Q_L^{\mathrm{RP}} \geq 0$. Heat extracted from L.

Adaptative strategy - Challenging the third law

If we choose adjust ω_d as $\omega_d \simeq \Omega_0 - T_L$:

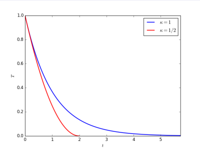
$$\frac{dT_L}{dt} = -\frac{1}{C_v}\dot{Q}_L$$

$$\dot{Q}_L^{\mathrm{RP}} = \frac{\pi^2}{8e}\frac{\delta\Omega^4}{\Omega_0^6} T_L I_L(T_L) \frac{I_R^0(\Omega_0)}{\Gamma_0} \propto \gamma_0 T_L^{1+\lambda_L}$$

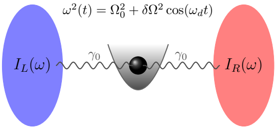
for $I_L(\omega) \propto \gamma_0\ \omega^{\lambda_L}$

Tipically, $C_v \propto T_L^d$, and therefore:

$$\frac{dT_L}{dt} = -\frac{1}{C_v}\dot{Q}_L \propto -\gamma_0 \eta T_L^{\overbrace{1 + \lambda_L - d}^{\kappa}}$$



Minimum temperature for a simple case



$$\dot{Q}_L^{\mathsf{RP}} = e^{-\frac{\Omega_0-\omega_d}{T_0}}\frac{|\delta\Omega|^4(\pi^2/8)}{\Omega_0^2(\Omega_0^2-(\Omega_0-\omega_d)^2)^2}$$
$$\times\frac{1}{\Gamma_0}\left\{I_R^0(\Omega_0)I_L^0(\Omega_0-\omega_d)(\Omega_0-\omega_d) - \xcancel{\Omega_0 I_L^0(\Omega_0)I_R^0(\Omega_0-\omega_d)}\right\}$$

- If $I_L(\omega) = I_R(\omega)$, then $Q_L^{\mathsf{RP}} \leq 0$. No symmetry broken.
- If $I_L(\Omega_0) \ll I_R(\Omega_0)$, then $Q_L^{\mathsf{RP}} \geq 0$. Heat extracted from L.

Adaptative strategy - Challenging the third law

If we choose adjust ω_d as $\omega_d \simeq \Omega_0 - T_L$:

$$\frac{dT_L}{dt} = -\frac{1}{C_v}\dot{Q}_L$$

$$\dot{Q}_L^{\mathsf{RP}} = \frac{\pi^2}{8e}\frac{\delta\Omega^4}{\Omega_0^6}T_L I_L(T_L)\frac{I_R^0(\Omega_0)}{\Gamma_0} \propto \gamma_0 T_L^{1+\lambda_L}$$

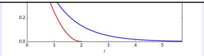
for $I_L(\omega) \propto \gamma_0\,\omega^{\lambda_L}$

Tip

Quantum Bath Refrigeration towards Absolute Zero: Challenging the Unattainability Principle

M. Kolář, D. Gelbwaser-Klimovsky, R. Alicki, and G. Kurizki
Phys. Rev. Lett. **109**, 090601 – Published 27 August 2012

$$\frac{\quad}{dt} = -\frac{\quad}{C_v}\dot{Q}_L \propto -\gamma_0\eta T_L$$

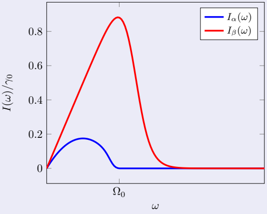


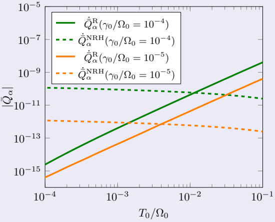
Minimum temperature for a simple case

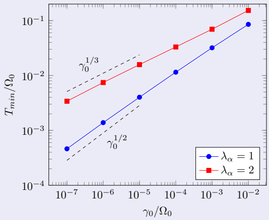
A simple scaling argument...

$$\begin{aligned} \dot{Q}_L^{\mathsf{RP}} &\propto \gamma_0 \, T_L^{1+\lambda_L} \\ \dot{Q}_L^{\mathsf{NRH}} &\propto \gamma_0^2 \end{aligned} \qquad \Rightarrow \qquad T_{\mathsf{min}} \propto \gamma_0^{\frac{1}{1+\lambda_L}}$$

A numerical verification...



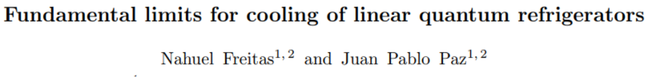




Main messages

- The dynamical Casimir effect is a fundamental limit for cooling in linear quantum refrigerators.
- This effect is non-resonant and next-to-leading order in the weak coupling approximation.
- The usual weak coupling approximation breaks down for low temperatures.

arXiv:1607.04234

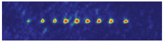


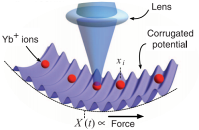
Fundamental limits for cooling of linear quantum refrigerators

Nahuel Freitas[1,2] and Juan Pablo Paz[1,2]

Next steps

- Study and design thermodynamical processes in ion crystals, nanomechanical devices, or circuit QED. Example: 'active' sympathetic cooling.





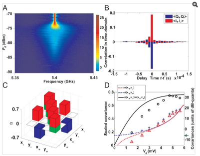
- Design strategies for the generation of steady state entanglement in open systems (quantum control?).

Bringing Entanglement to the High Temperature Limit

Fernando Galve, Leonardo A. Pachón, and David Zueco
Phys. Rev. Lett. **105**, 180501 – Published 25 October 2010

Dynamical Casimir effect in a Josephson metamaterial

Pasi Lähteenmäki[a,1], G. S. Paraoanu[a], Juha Hassel[b], and Pertti J. Hakonen[a]



- Extend results to the non-linear regime.

Cooling processes and minimum temperature

Design of cooling processes

$$\dot{Q}_\alpha = \dot{Q}_\alpha^{\text{RP}} + \overbrace{\dot{Q}_\alpha^{\text{RH}} + \dot{Q}_\alpha^{\text{NRH}}}^{\leq 0}$$

For cooling, we need $\dot{Q}_\alpha^{\text{RP}} > 0$.

Two general strategies:

- Spatial symmetry, asymmetric driving (directional pumping)
- Spatial asymmetry, symmetric driving (heat rectification)

Minimum temperature

Optimistic setting:

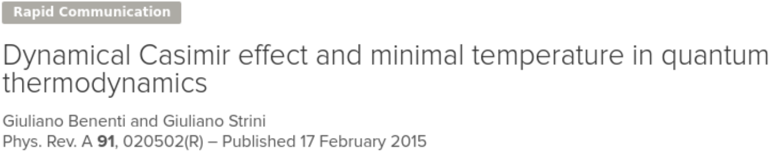
- Equal temperatures ($T_\alpha = T_0$)
- $\dot{Q}_\alpha^{\text{RP}} + \dot{Q}_\alpha^{\text{RH}} \geq 0$ for all T_0

Still, there exists a value of T_0 below which:

$$\dot{Q}_\alpha^{\text{RH}} < |\dot{Q}_\alpha^{\text{NRH}}|$$

This value of T_0 is the minimum temperature T_{min} (not universal, depends on the process)

Some similar results



Rapid Communication

Dynamical Casimir effect and minimal temperature in quantum thermodynamics

Giuliano Benenti and Giuliano Strini
Phys. Rev. A **91**, 020502(R) – Published 17 February 2015