Quantum Speed Limits

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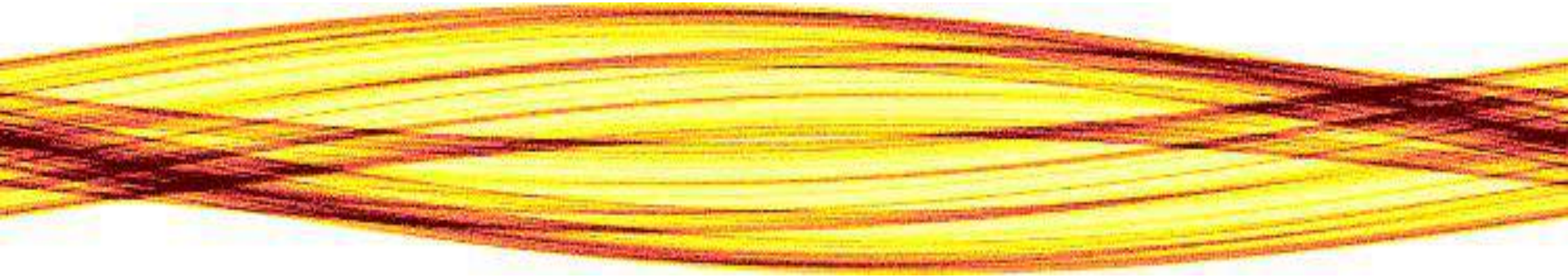

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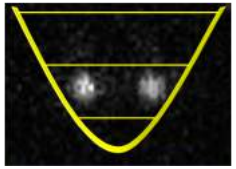
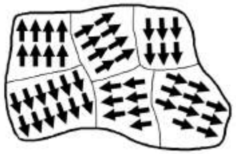
XVIII Giambiagi Winter School: Quantum Chaos & Control

July 25-29 2016, Buenos Aires



Talk 1: STA in noncritical systems



Talk 2: STA in critical systems

Talk 3: Quantum Speed Limits

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Talk 2: Contents

The Kibble-Zurek mechanism

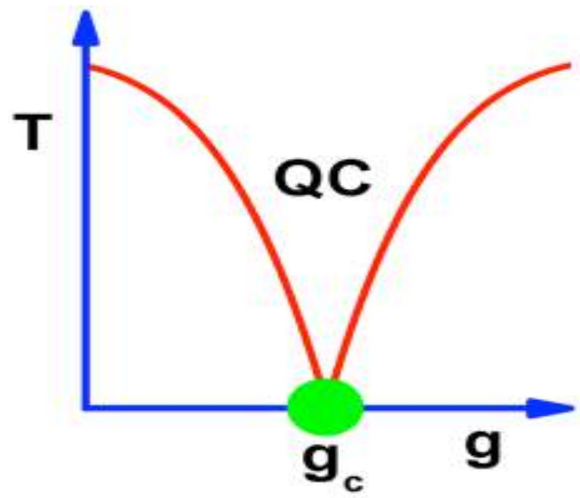
- Universal phase-transition dynamics
- Quantum Annealing

Ways out

- Inhomogeneous driving
- Counterdiabatic driving

Idea II

Counterdiabatic driving in a Quantum Phase Transition



Example: 1d Quantum Ising Chain

Ising chain Hamiltonian $$\hat{H}_0(t) = -\sum_{n=1}^{N} \left[\sigma_n^x \sigma_{n+1}^x + g(t)\sigma_n^z\right]$$

Critical point $g_c = 1$

$g \gg 1$ $|\rightarrow\rightarrow\rightarrow\cdots\rightarrow\rangle$

z-axis

$g \ll 1$ $|\uparrow\uparrow\uparrow\ldots\uparrow\rangle$

$|\downarrow\downarrow\downarrow\ldots\downarrow\rangle$

x-axis

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x-axis

Excitations: $|\dots \uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow \dots\rangle$

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x-axis

Excitations: $|\ldots \uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow \ldots\rangle$

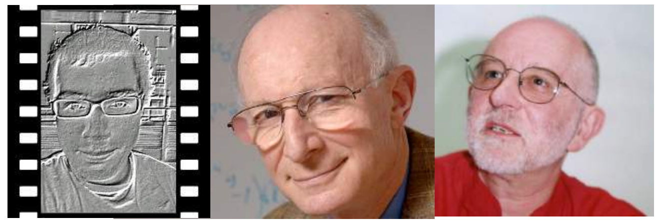
Kibble-Zurek Mechanism

$$n_{ex} \propto \frac{1}{\sqrt{\tau_Q}}$$

Ways out of the Kibble-Zurek mechanism?

Counterdiabatic driving: Ising Chain

Counterdiabatic driving?



Auxiliary control

$$\hat{H}_1(t) = i\hbar \sum_n \left(|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle |n\rangle\langle n|\right)$$

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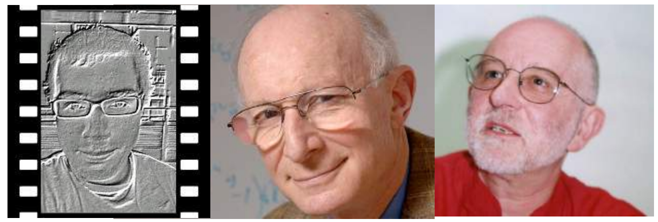
A. del Campo, M. M. Rams, W. H. Zurek, PRL 109, 115703 (2012)

Counterdiabatic driving: Ising Chain

Counterdiabatic driving?



Auxiliary control

$$\hat{H}_1(t) = i\hbar \sum_n \left(|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle |n\rangle\langle n|\right)$$

Diagonalization: Jordan Wigner transformation + Fourier transform

$$\hat{H}_0(t) = 2\sum_{k>0} \Psi_k^\dagger \left[\sigma_k^z (g(t) - \cos k) + \sigma_k^x \sin k\right] \Psi_k$$

$$\hat{H}_1(t) = -\dot{g}(t) \sum_{k>0} \frac{1}{2} \frac{\sin k}{g^2 + 1 - 2g\cos k} \Psi_k^\dagger \sigma_k^y \Psi_k$$

→ Long-range many-body interaction!

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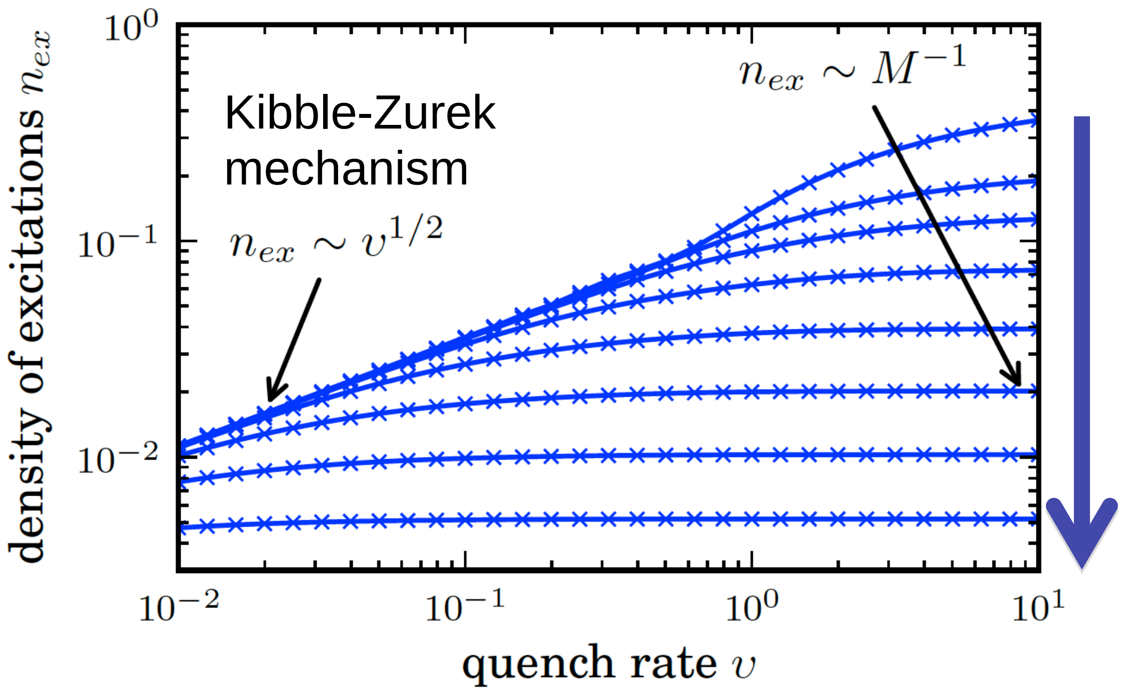
Truncated Auxiliary Hamiltonian

Quench through the critical point $g(t) = g_c - vt$

Truncated Auxiliary Hamiltonian



Range of auxiliary Hamiltonian

Suppressing Kibble-Zurek mechanism

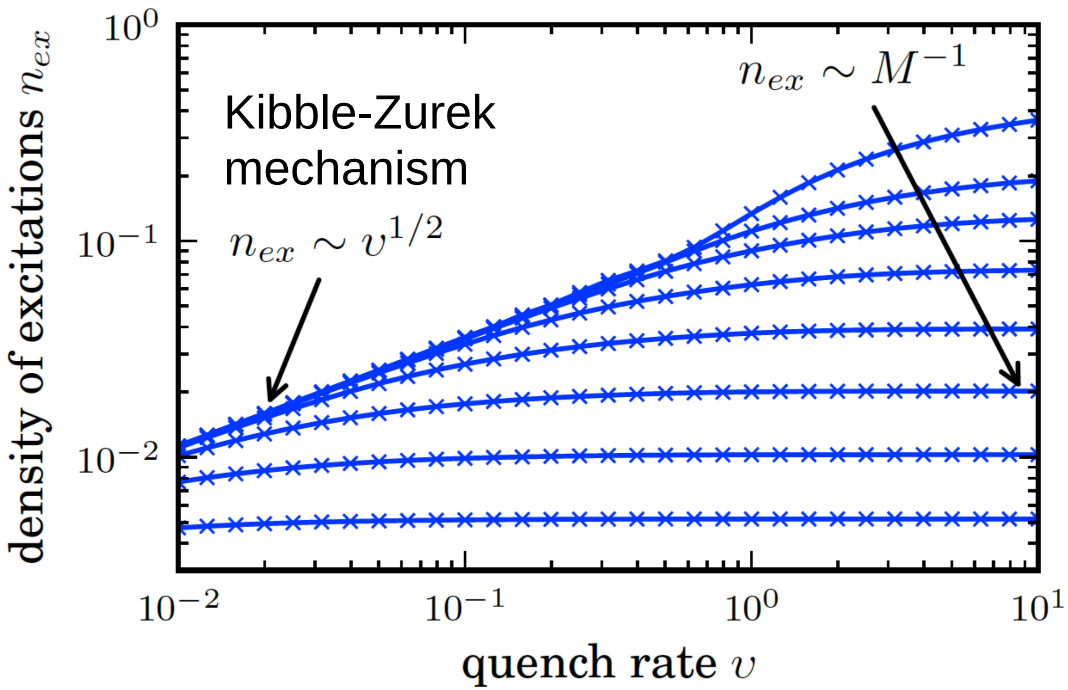
A. del Campo, M. M. Rams, W. H. Zurek, PRL 109, 115703 (2012)

Truncated Auxiliary Hamiltonian

Quench through the critical point $g(t) = g_c - vt$

Truncated Auxiliary Hamiltonian

Experimentalist's view



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A. del Campo, M. M. Rams, W. H. Zurek, PRL 109, 115703 (2012)

Tailoring control fields

H. Saberi, T. Opatrný, K. Mølmer, AdC, Phys. Rev. A 90, 060301(R) (2014)

Tailoring auxiliary interactions

Set of available controls $\{L_k\}$

Approximated Auxiliary Hamiltonian $\tilde{H}_1 = \sum_{k=1}^{K} \alpha_k L_k$

Minimize the norm $\min_{\{\alpha_k\}} ||(H_1 - \tilde{H}_1)|GS(t)\rangle||^2$

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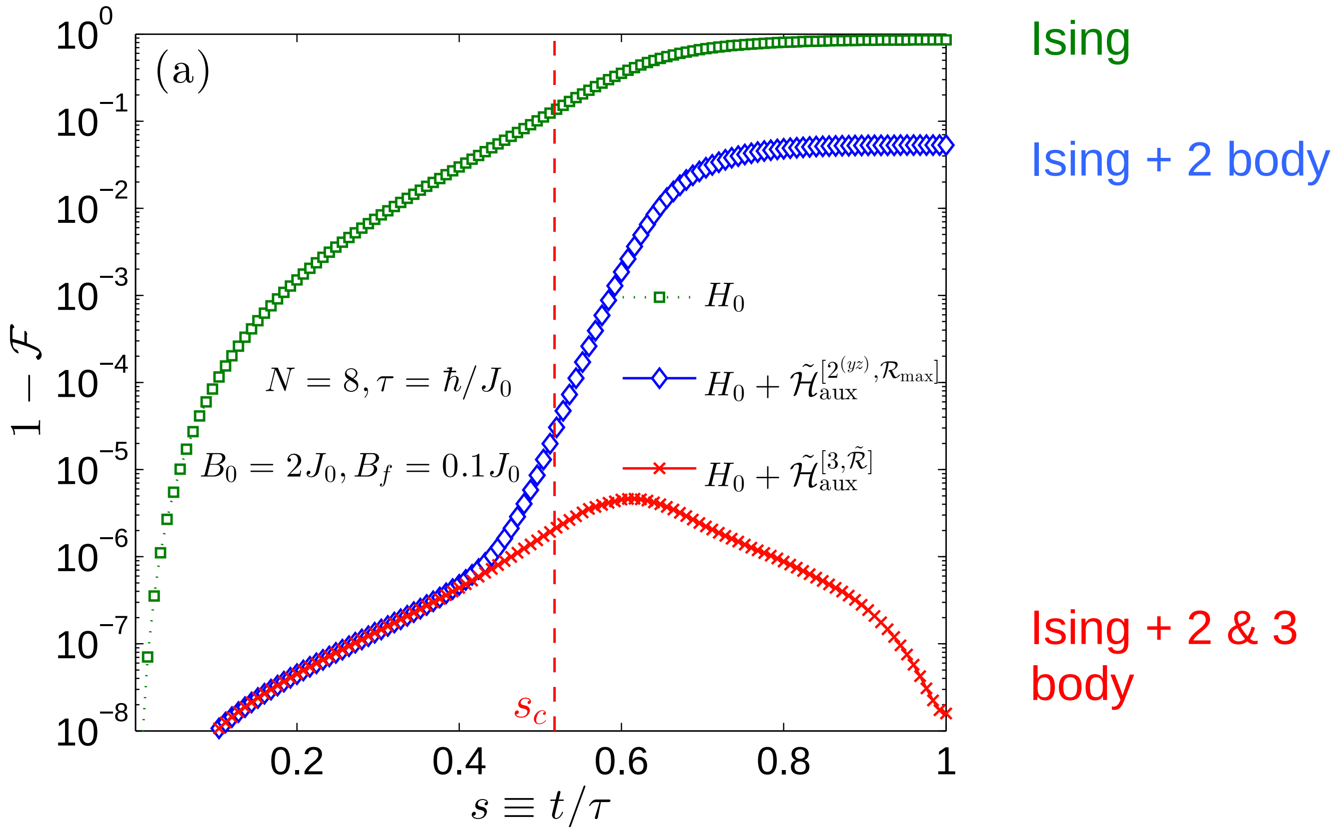
T. Opatrný, K. Mølmer, NJP 16, 015025 (2014)
H. Saberi, T. Opatrný, K. Mølmer, AdC, Phys. Rev. A 90, 060301(R) (2014)

Suppressing KZM/excitations

Approximated Auxiliary Hamiltonian $$\tilde{H}_1(t) = \sum_{i_1,i_2}{}' h^{y,z}_{i_1,i_2}(t)\, \sigma^y_{i_1} \otimes \sigma^z_{i_2}$$



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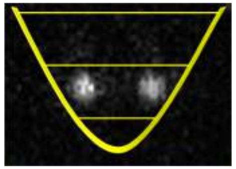
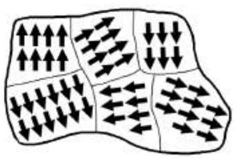


Talk 1: STA in noncritical systems



Talk 2: STA in critical systems

Talk 3: Quantum Speed Limits

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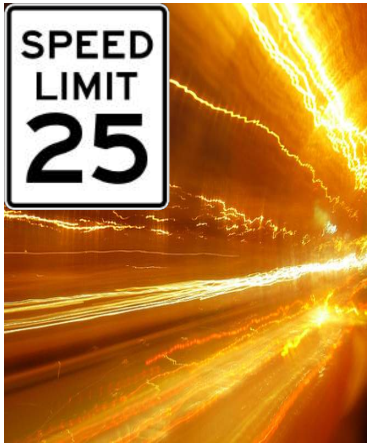
Ultimate Quantum Speed Limits

Ultimate Quantum Speed Limits

Courtesy of Guy Chenu

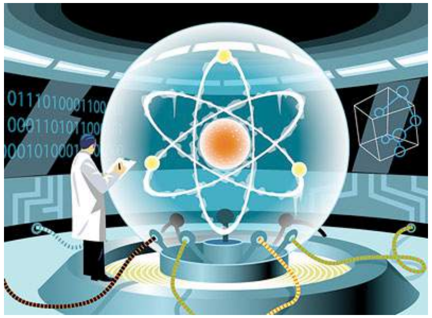
Speed limits for Isolated systems



How fast can we go?

Not faster than the Quantum Speed Limit

Minimum time required for a quantum state to evolve to an orthogonal state



$$T \geq \frac{\pi}{2} \max\left(\frac{\hbar}{E}, \frac{\hbar}{\Delta E}\right)$$

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Time-energy uncertainty relation

Beautiful history

Passage time: Minimum time required for a state to reach an orthogonal state

Landau

Krylov

1945 Mandelstam and Tamm "MT"

$$\tau \geq \frac{\pi}{2}\frac{\hbar}{\Delta H}$$

1967 Fleming

1990 Anandan, Aharonov

1992 Vaidman, Ulhman

1993 Uffnik

1998 Margolus & Levitin "ML"

2000 Lloyd

2003 Giovannetti, Lloyd, Maccone: MT & ML unified

2003 Bender: no bounds in PT-symmetric QM

2009 Levitin,Toffoli

2013 2013 Bound for open (as well as unitary) system dynamics!

AdC, I.L. Egusquiza, M. B. Plenio, S. Huelga PRL **110**, 050403 (2013)

Two seminal results

Mandelstam-Tamm (1945)

Heisenberg EOM + definition of time = TEUR

$$\frac{d}{dt}\hat{A} = \frac{1}{i\hbar}[\hat{A}, \hat{H}] \qquad \tau(A) = \frac{\Delta A}{\frac{d}{dt}\langle \hat{A} \rangle}$$

$$\tau(A)\Delta H \geq \frac{\hbar}{2}$$

Margolus-Levitin (1998)

Survival amplitude, vanishing real and imaginary parts

$$|\Psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar}|n\rangle \quad a(t) := \langle \psi(0)|\psi(t)\rangle \quad \mathrm{Re}(a) \geq 1 - \frac{2E}{\pi\hbar}t + \mathrm{Im}(a)$$

$$\tau \geq \frac{\hbar\pi}{2E}$$

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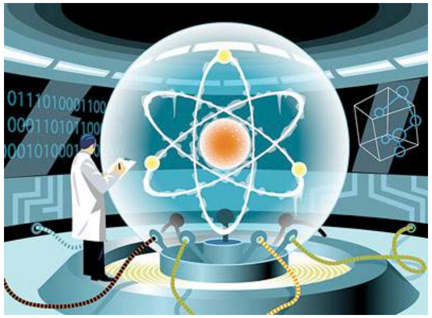
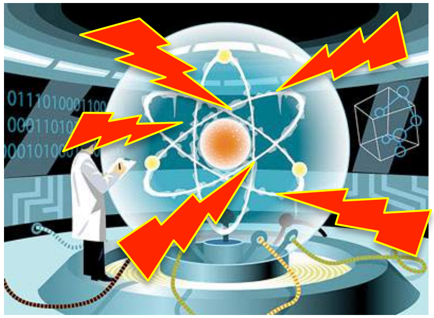
Speed limits for arbitrary systems



Isolated **systems: unitary dynamics**

$$T \geq \frac{\pi}{2}\max\left(\frac{\hbar}{E}, \frac{\hbar}{\Delta E}\right)$$

Real **systems: coupled to an environment**



Nonunitary dynamics (master equation) $$\frac{d\rho_t}{dt} = \mathcal{L}\rho_t$$

What replaces energy in an open system?

Bound to the speed of evolution for open (as well as unitary) system dynamics

$$f(t) = \mathrm{tr}(\rho_0\rho_t) \quad |\dot{f}(t)| = |\mathrm{tr}(\rho_0\mathcal{L}\rho_t)| \leq \sqrt{\mathrm{tr}[(\mathcal{L}^\dagger\rho_0)^2]}$$

includes coupling to an environment

$$E, \Delta E \rightarrow \sqrt{\mathrm{Tr}[(\mathcal{L}^\dagger\rho_0)^2]}$$

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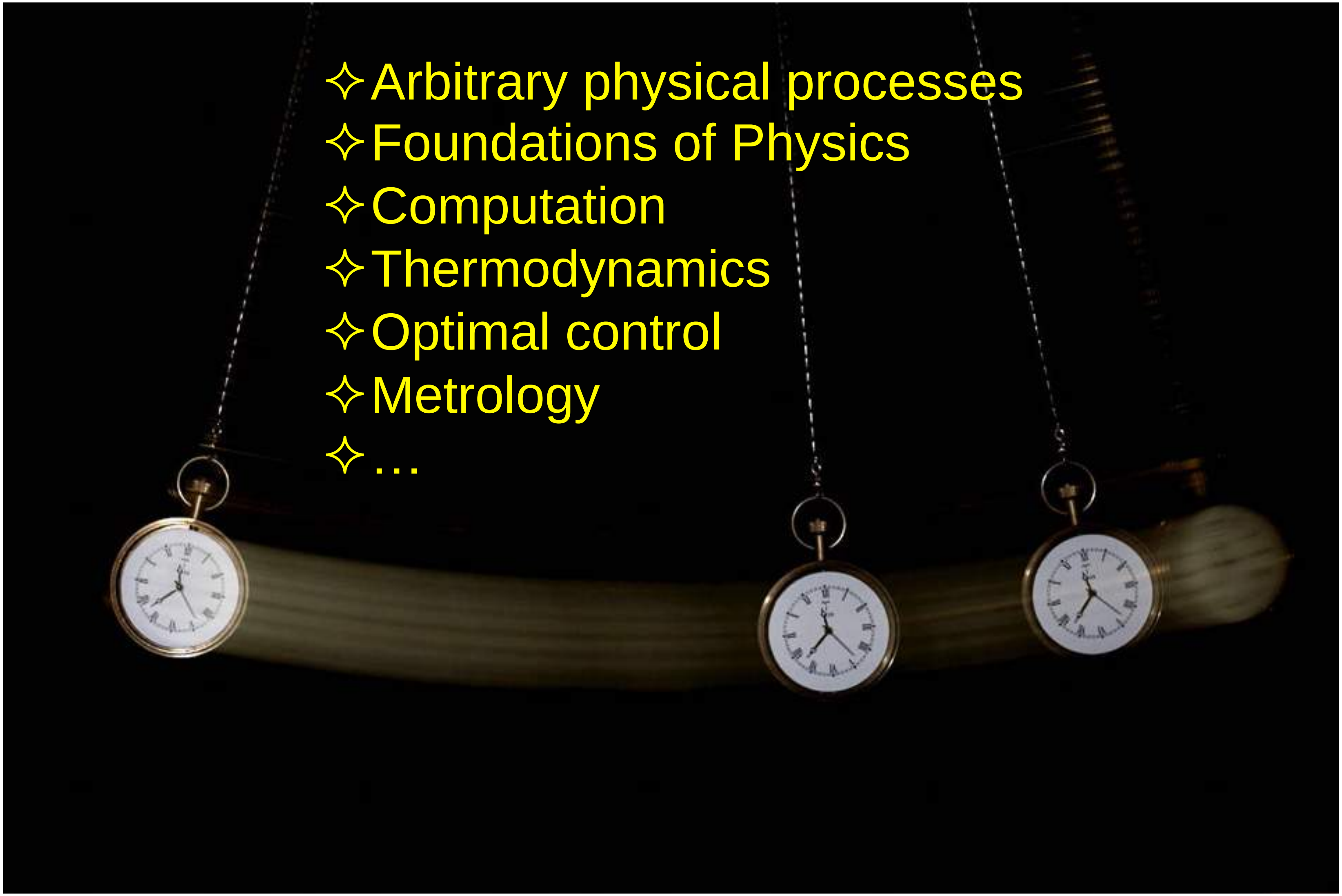
AdC et al. PRL **110**, 050403 (2013)
Taddei et al. PRL **110**, 050402 (2013)
See too Deffner et Lutz, PRL **111**, 010402 (2013)

Applications of Quantum Speed Limits



- ✧Arbitrary physical processes
- ✧Foundations of Physics
- ✧Computation
- ✧Thermodynamics
- ✧Optimal control
- ✧Metrology
- ✧…

Courtesy of Guy Chenu

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QSL to the generation of Quantumness

Coherence monotone $$Q(t) = 2\|[\rho_0, \rho_t]\|^2 \in [0,1]$$

Arbitrary physical process $$\frac{d\rho_t}{dt} = \mathcal{L}\rho_t$$

Quantum Speed Limit $$\tau \geq \sqrt{\frac{Q(\tau)}{2}} \frac{1}{\overline{\|[\rho_0, \mathcal{L}\rho_t]\|}}$$

Different in nature from MT & ML bounds

Jing, Wu, AdC, arXiv:1510.01106

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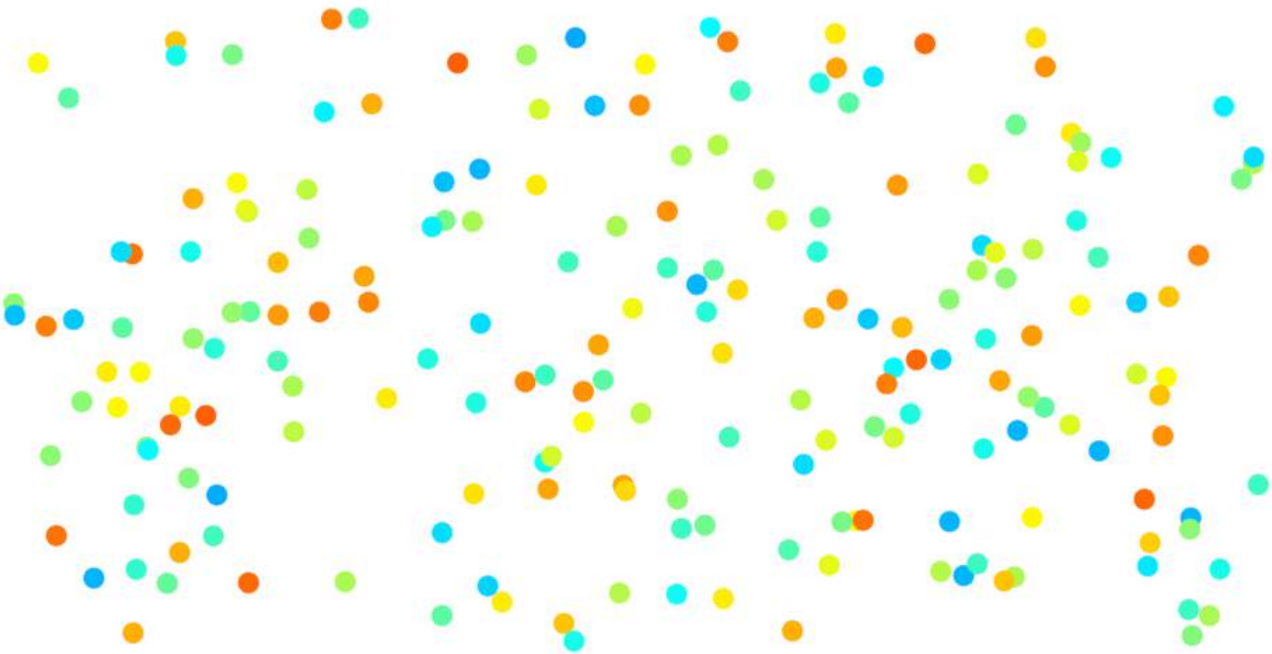
Applications
Quantum Speed Limits in finite-time thermodynamics



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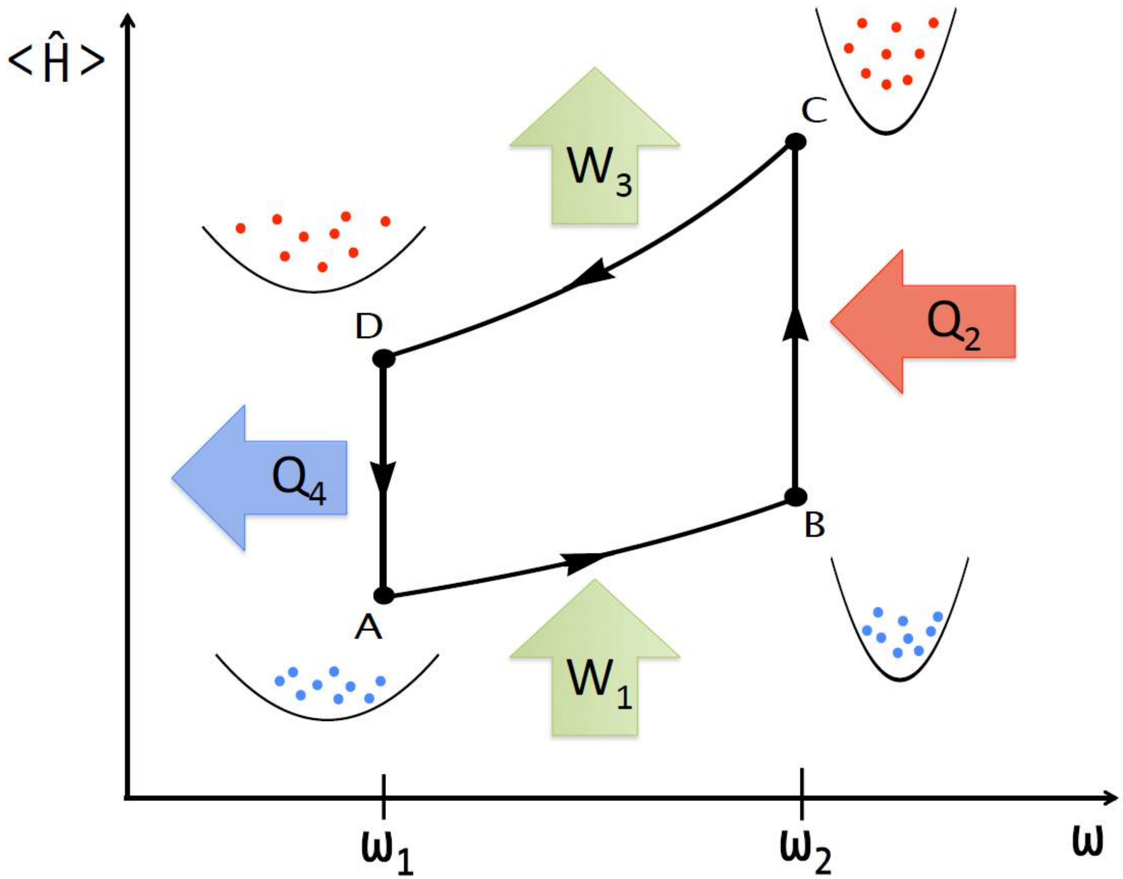
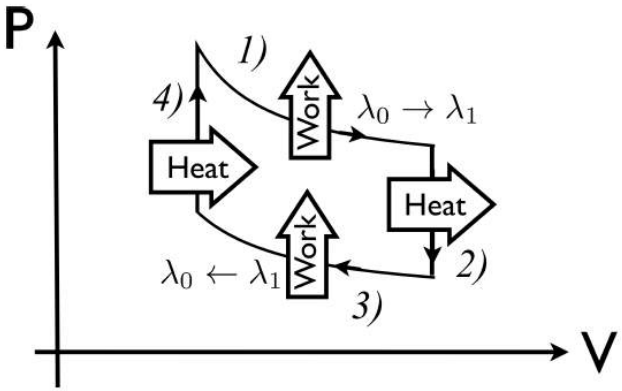
Quantum Heat Engines (e.g. Otto Cycle)



Efficiency vs Power



Quantum efficiency

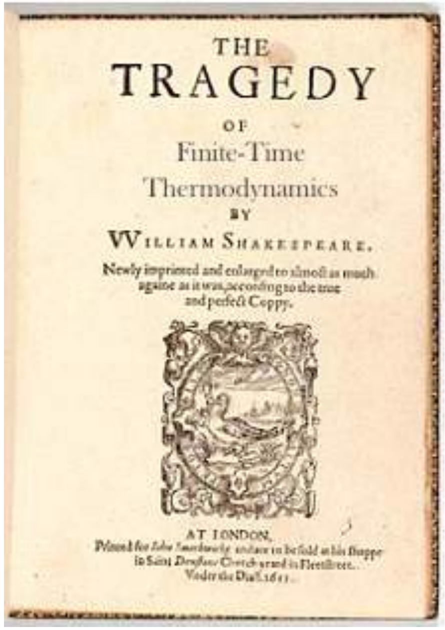
$$\eta = -\frac{\langle W_1\rangle + \langle W_3\rangle}{\langle Q_2\rangle}$$

Nonadiabatic Efficiency e.g. of a single-particle Otto cycle

$$\eta = 1 - \frac{\omega_1}{\omega_2} f[\omega(t)] \leq 1 - \frac{\omega_1}{\omega_2}$$

Essence of finite-time thermodynamics:

Trade-off between efficiency and power



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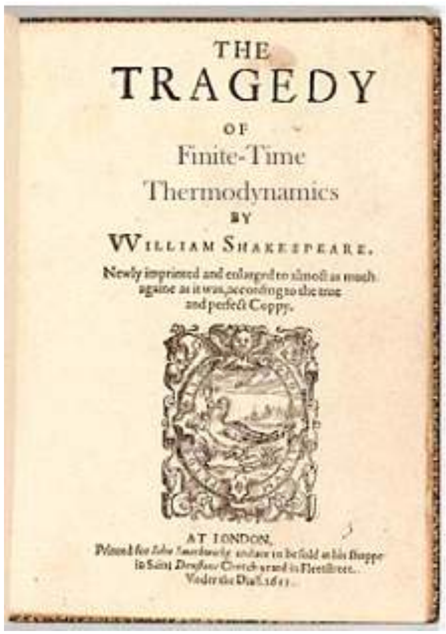
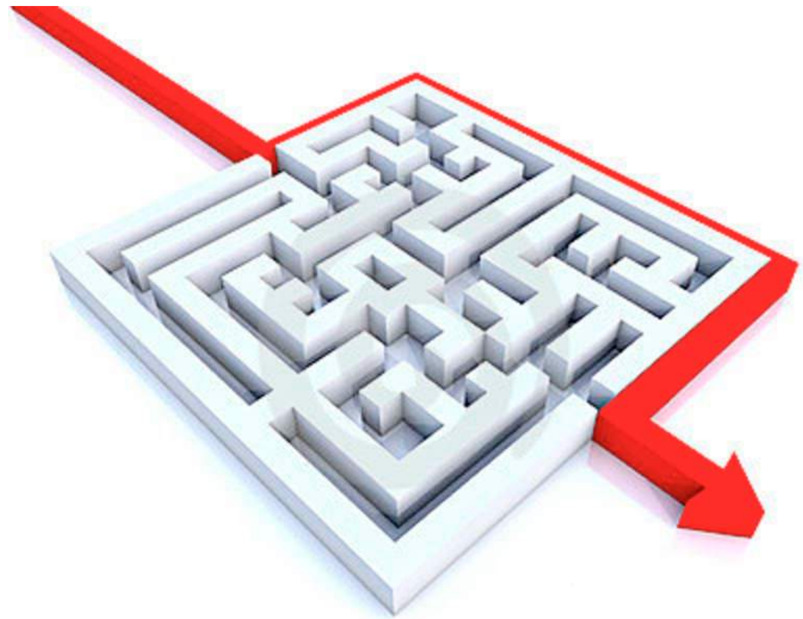
Shortcuts as a way out of the tragedy



Shortcuts to adiabaticity



- AdC, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014) (single-particle)
- M. Beau, J. Jaramillo, AdC, Entropy 18, 168 (2016) (many-particle)

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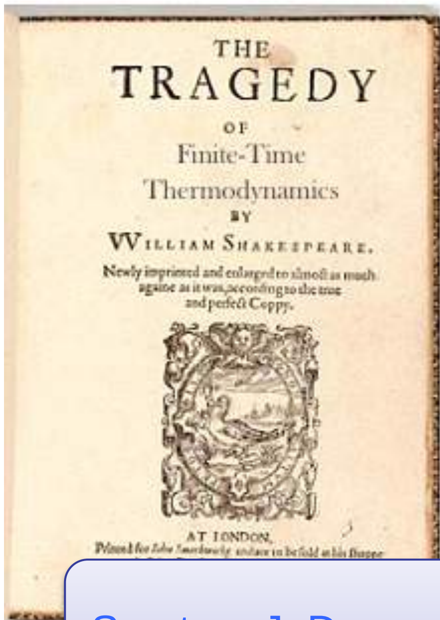
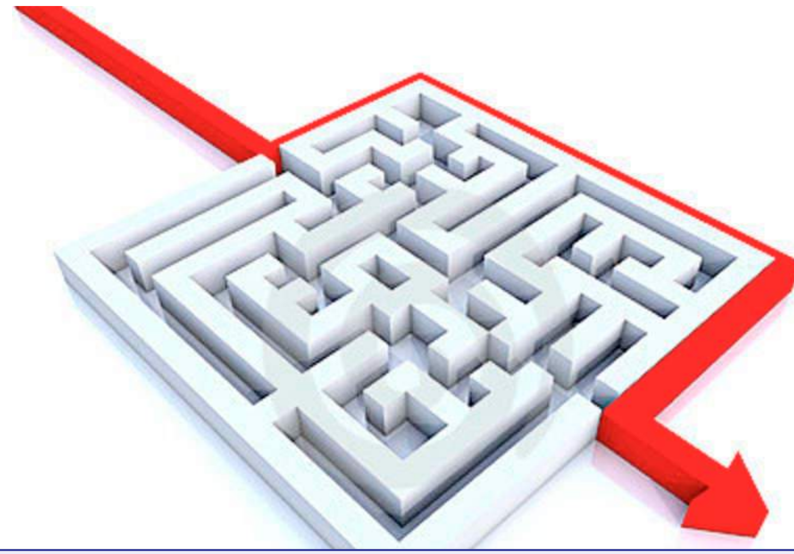
Shortcuts as a way out of the tragedy



Shortcuts to adiabaticity



See too: J. Deng et al., Phys. Rev. E **88**, 062122 (2013) (single-particle)

- AdC, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014) (single-particle)
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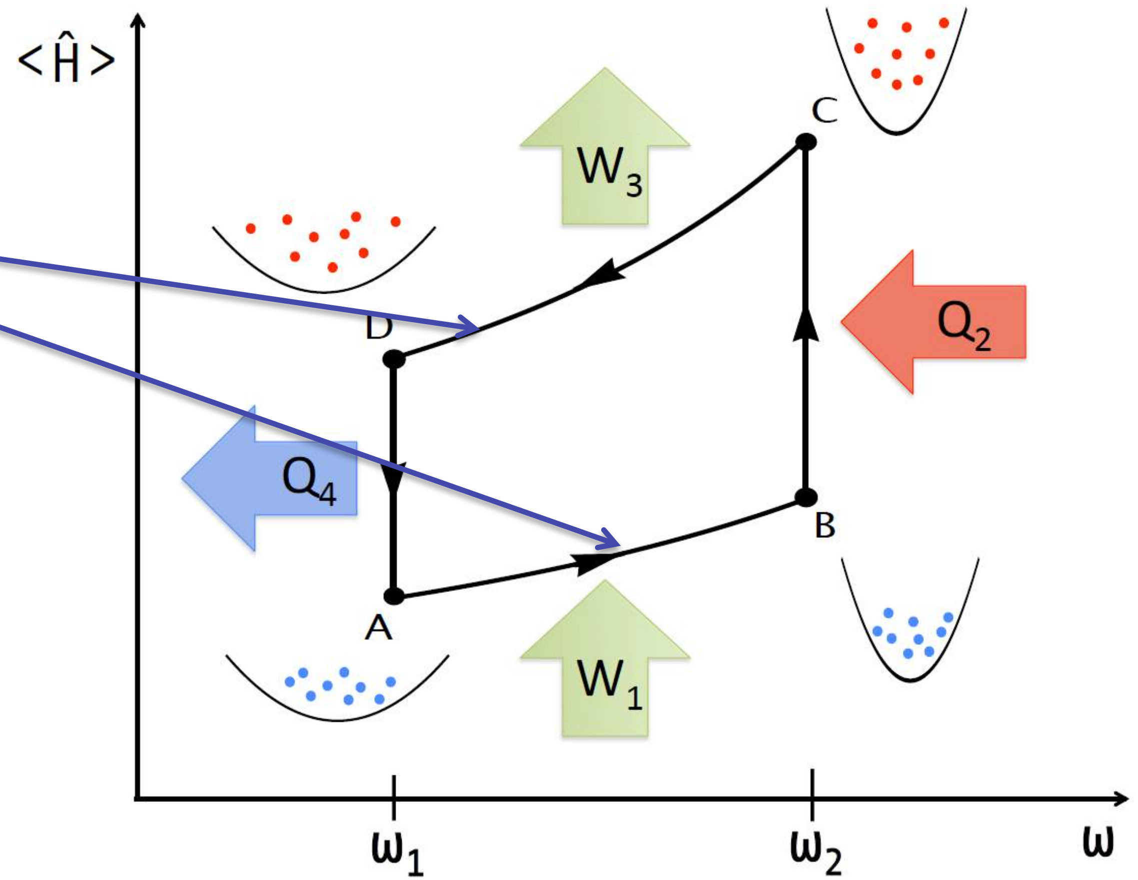
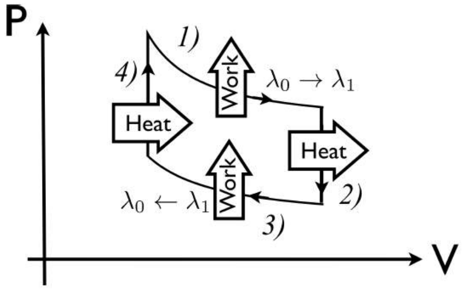
Quantum Heat Engines (e.g. Otto Cycle)

STA to expansion and compression



Superadiabatic quantum engine

STA in steps 1 & 2
Thermalization time << adiabats

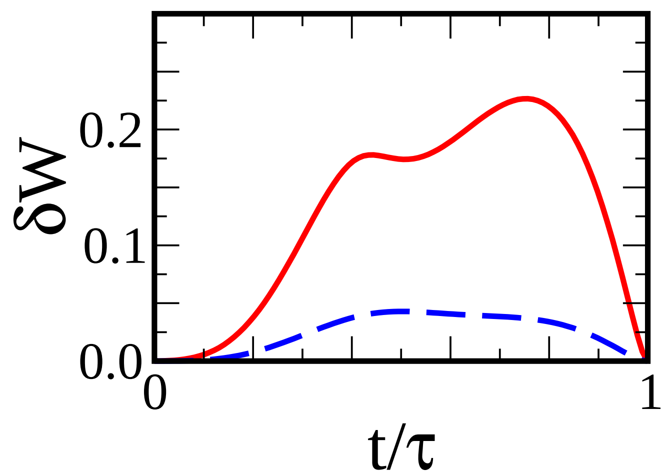


Thermodynamic cycle at finite power and zero friction
i.e., maximum efficiency

$$\mathcal{E}_{\max} = 1 - \frac{\omega(\tau)}{\omega(0)}$$

Thermal state at t=0 in stroke 1)

$$\delta W = \langle W \rangle - \langle W_{\rm ad} \rangle = \frac{1}{\beta_t} S(\rho_t || \rho_t^{\rm ad}), \quad \beta_t = \beta_0 \epsilon_n(0)/\epsilon_n(t)$$



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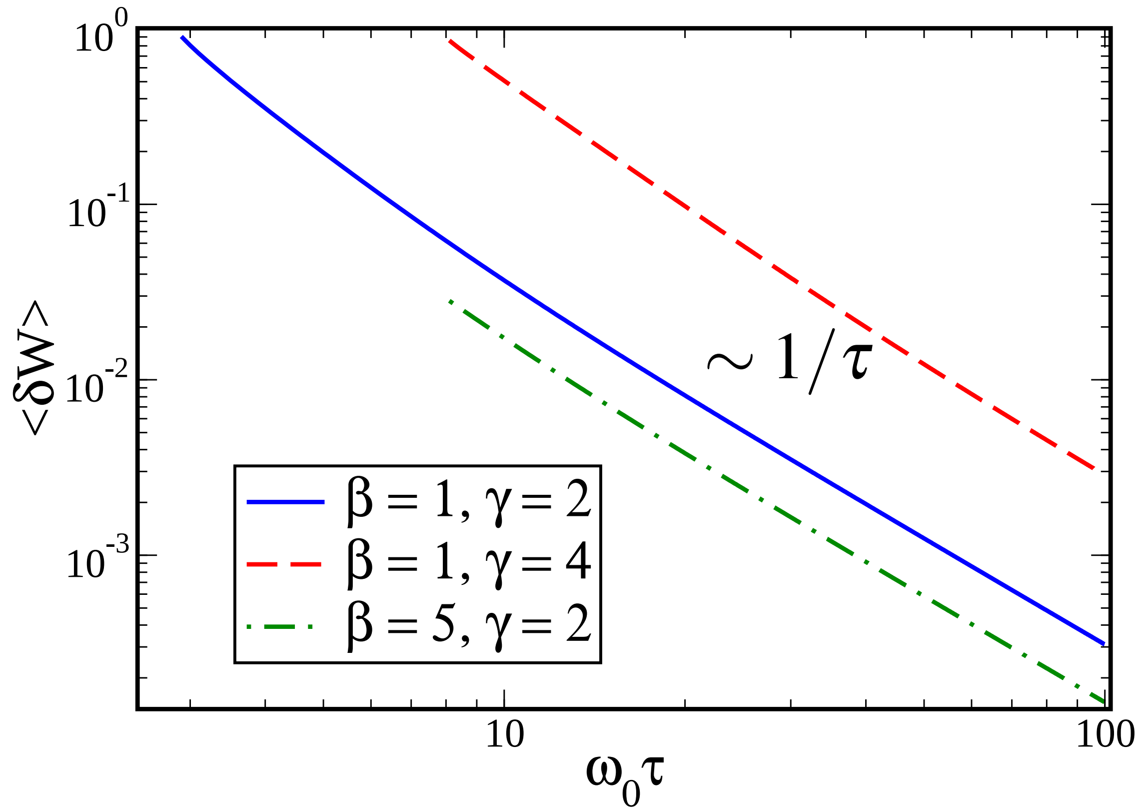
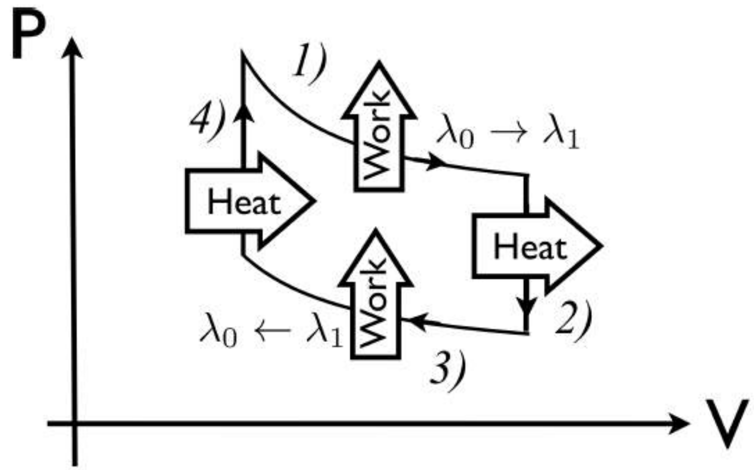
Energy Cost of Shortcuts to Adiabaticity

$$\delta W = \langle W \rangle - \langle W_{\rm ad} \rangle \qquad \langle \delta W \rangle = \frac{1}{\tau}\int_0^\tau \delta W dt$$



Performance: Ultimate Quantum Limits



Fundamental bound to the output power of a QHE

$$\mathscr{P} \leq -\frac{\langle W_{\mathrm{ad},1}(\tau)\rangle + \langle W_{\mathrm{ad},3}(\tau)\rangle}{\hbar \mathscr{L}\left(\rho_\tau^{\mathrm{eq}}, \rho_0\right)} \max\{E_\tau, \Delta E_\tau\}$$

To be extended to include strokes with open dynamics

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AdC, Goold, Paternostro, Sci. Rep. **4**, 6208 (2014)

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Scalable QHE assisted by shortcuts to adiabaticity

- **Thermodynamic cycle working at tunable finite power and zero friction**

- **Quantum speed limits impose ultimate performance bounds**

Eco-friendly Lamborghini!

- AdC, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014)
- M. Beau, J. Jaramillo, AdC, Entropy **18**, 168 (2016)

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Applications

Quantum Speed Limits
and counterdiabatic driving

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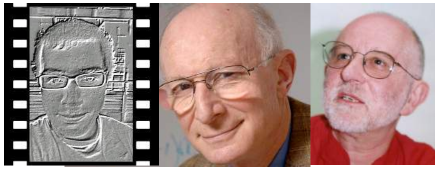
Energy cost of counterdiabatic driving



Full Driving Hamiltonian $H[\lambda(t)] = H_0[\lambda(t)] + H_1[\lambda(t)]$

Auxiliary control (with no degeneracy)

$$H_1 = i\hbar\dot{\lambda}\sum_n \sum_{k\neq n} |n\rangle \frac{\langle n|\partial_\lambda|k\rangle}{E_k - E_n}\langle k|$$

Energy fluctuations

$$\Delta H^2 = \langle 0(t)|H_1^2|0(t)\rangle = |\dot{\lambda}|^2 \chi_F(\lambda)$$

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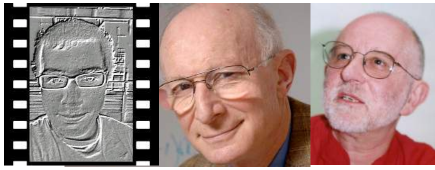
A. del Campo, M. M. Rams, W. H. Zurek, PRL 109, 115703 (2012)

Energy cost of counterdiabatic driving



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Energy fluctuations

$$\Delta H^2 = \langle 0(t)|H_1^2|0(t)\rangle = |\dot{\lambda}|^2\chi_F(\lambda)$$

Fidelity susceptibility

$$|\langle 0(\lambda)|0(\lambda+\delta)\rangle|^2 \approx 1 - \delta^2\chi_F(\lambda)$$

$$\chi_F(\lambda) = \sum_{n\neq 0}\frac{|\langle n(\lambda)|\partial_\lambda H_0|0(\lambda)\rangle|^2}{(E_n - E_0)^2}$$

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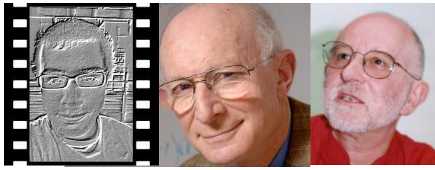

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Energy fluctuations

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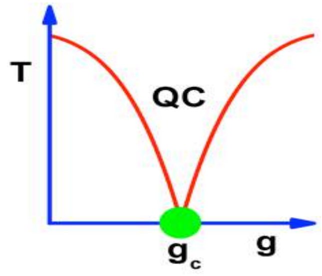
Fidelity susceptibility across a phase transition: divergence with system size



$$\chi_F(\lambda) \sim N|\lambda - \lambda_c|^{d\nu-2}$$

$$\chi_F(\lambda_c) \sim N^{2/d\nu}$$

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AdC, M. M. Rams, W. H. Zurek, PRL **109**, 115703 (2012)
H. Saberi, T. Opatrný, K. Mølmer, AdC, Phys. Rev. A **90**, 060301(R) (2014)
AdC & K. Sengupta, Eur. Phys. J. Special Topics **224**, 189 (2015)

Summary

Shortcuts to adiabaticity speed up processes by tailoring excitations

Yet, Quantum Speed Limits rule any dynamical process

- Mandelstam-Tamm, Margolus-Levitin, QSL for open systems
- QSL in Quantum Thermodynamics
- QSL to counterdiabatic driving

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The Group

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Thanks for your attention!!

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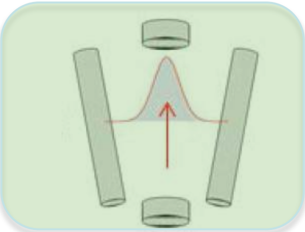
MSc, PhD students & postdocs!

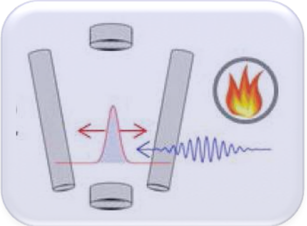
Time-energy uncertainty relation

***Isolated* systems: unitary dynamics**



$$T \geq \frac{\pi}{2}\max\left(\frac{\hbar}{E-E_0},\frac{\hbar}{\Delta E}\right)$$

***Real* systems: coupled to an environment**



Nonunitary dynamics $$\frac{d\rho_t}{dt}=\mathcal{L}\rho_t$$

What is the speed limit in an open system?

Bound to the speed of evolution for open (as well as unitary) system dynamics

includes coupling to an environment $$E,\Delta E \rightarrow \sqrt{\mathrm{Tr}[(\mathcal{L}^{\dagger}\rho_0)^2]}$$

AdC, I.L. Egusquiza, M. B. Plenio, S. Huelga PRL **110**, 050403 (2013)
See too: M. M. Taddei et al. PRL **110**, 050402 (2013)
S. Deffner and E. Lutz, PRL **111**, 010402 (2013)

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Quantum Heat Engine: Otto cycle