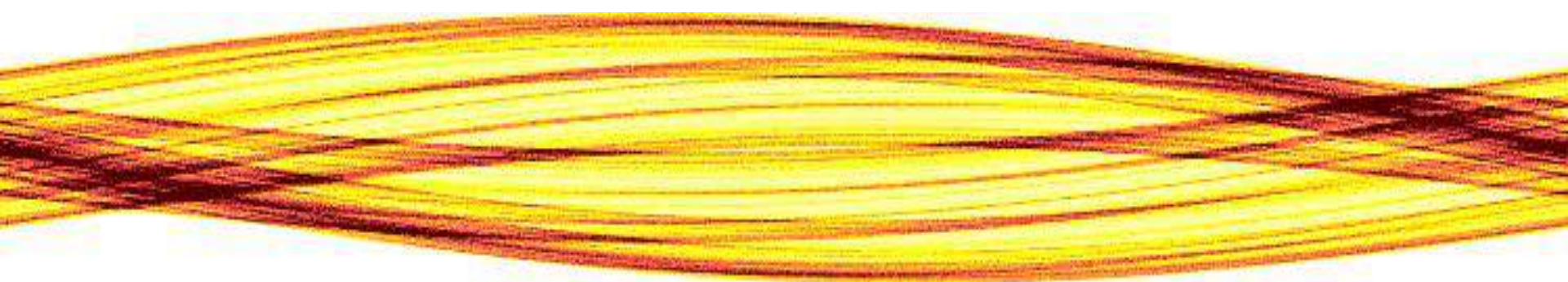


# Shortcuts in critical systems

*Rhapsody on a theme of Kibble and Zurek*

Adolfo del Campo

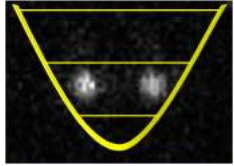
Department of Physics  
University of Massachusetts, Boston



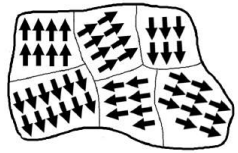
XVIII Giambiagi Winter School: Quantum Chaos & Control

July 25-29 2016, Buenos Aires





Talk 1: STA in noncritical systems



Talk 2: STA in critical systems



Talk 3: Quantum Speed Limits

# Talk 2: Contents

---

## The Kibble-Zurek mechanism

- ◆ Universal phase-transition dynamics
- ◆ Quantum Annealing

## Ways out

- ◆ Inhomogeneous driving
- ◆ Counterdiabatic driving

# Adiabatic dynamics

*Slow driving of a system*

*Provides good control*

*No excitations*

*Ground state*

*So?*



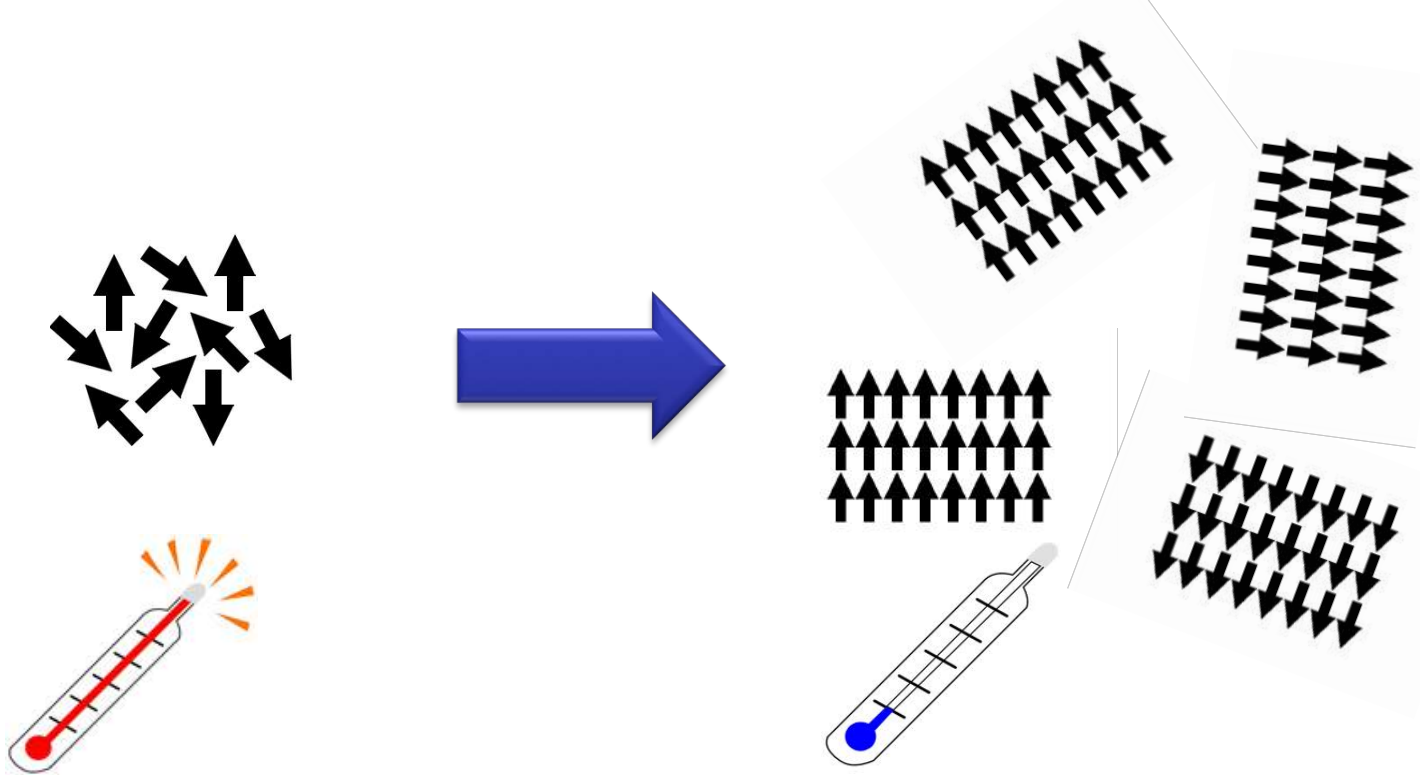
# Breaking symmetries

Breaking adiabatic dynamics  
in critical systems



# Spontaneous symmetry breaking

Driving through a phase transition  
(e.g. paramagnetic-ferromagnetic transition)

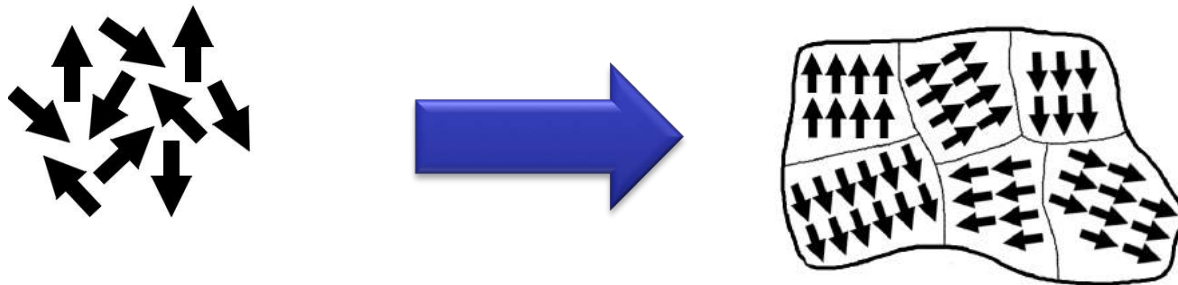


degenerate ground states

# Spontaneous symmetry breaking

Driving through a phase transition  
(e.g. paramagnetic-ferromagnetic transition)

Cooling at finite rate



Broken symmetry  
Size of the domains?  
How many?  
The Kibble-Zurek mechanism



# Tom W. B. Kibble



Madras - London  
June 3<sup>rd</sup> 2016

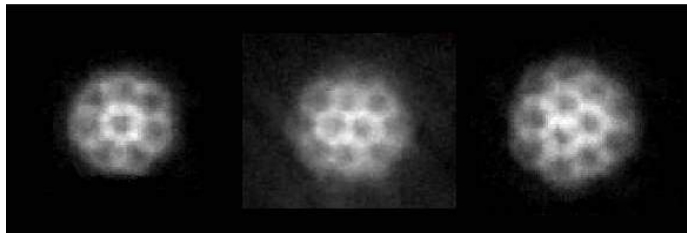


T. W. B. Kibble, JPA 9, 1387 (1976)

T. W. B. Kibble, Phys. Rep. 67, 183 (1980)



# Wojciech H. Zurek



W. H. Zurek, *Nature* (London) 317, 505 (1985)

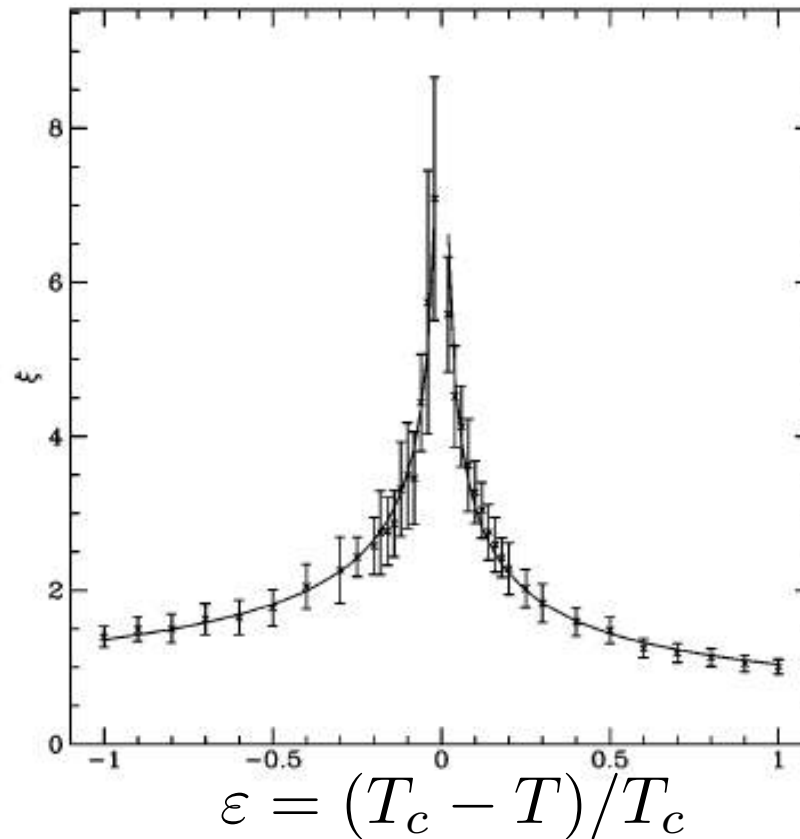
W. H. Zurek, *Acta Phys. Pol. B.* 1301 (1993)

# Scaling theory near critical point

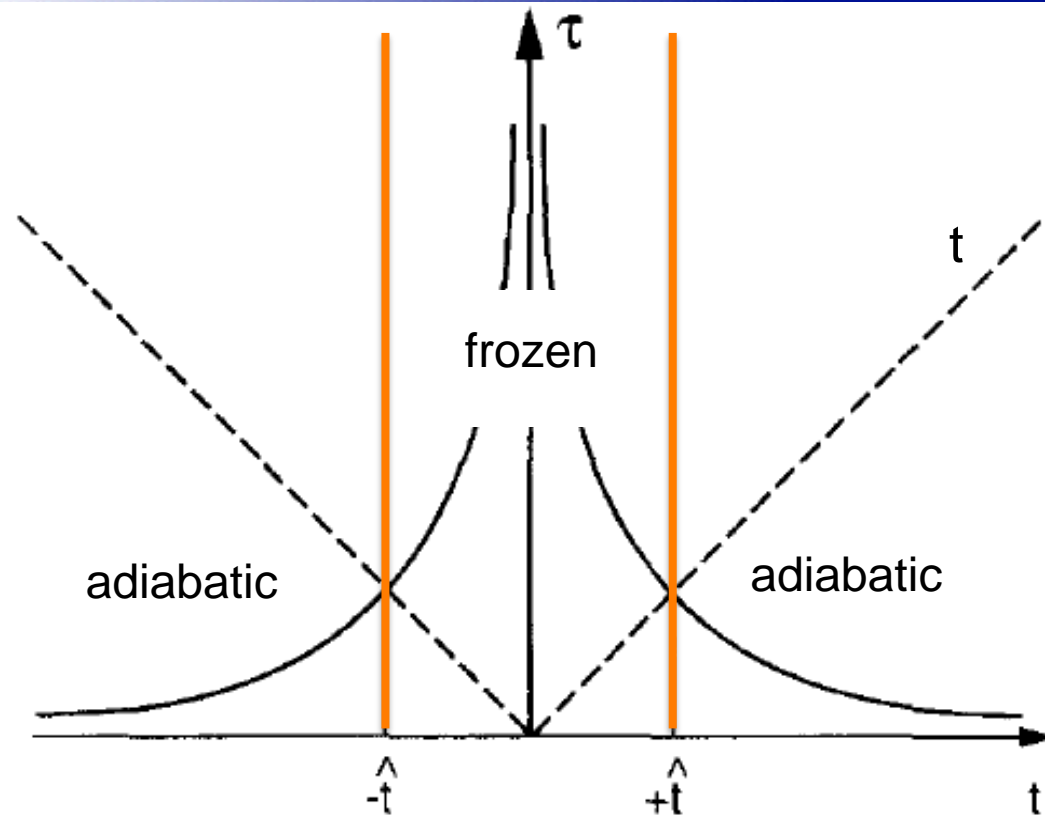
Universal divergence of

Correlation length  $\xi(t) = \frac{\xi_0}{|\varepsilon(t)|^\nu}$

Relaxation time  $\tau(t) = \frac{\tau_0}{|\varepsilon(t)|^{z\nu}}$

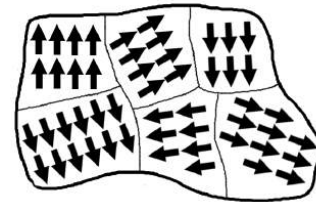


# The Kibble-Zurek mechanism

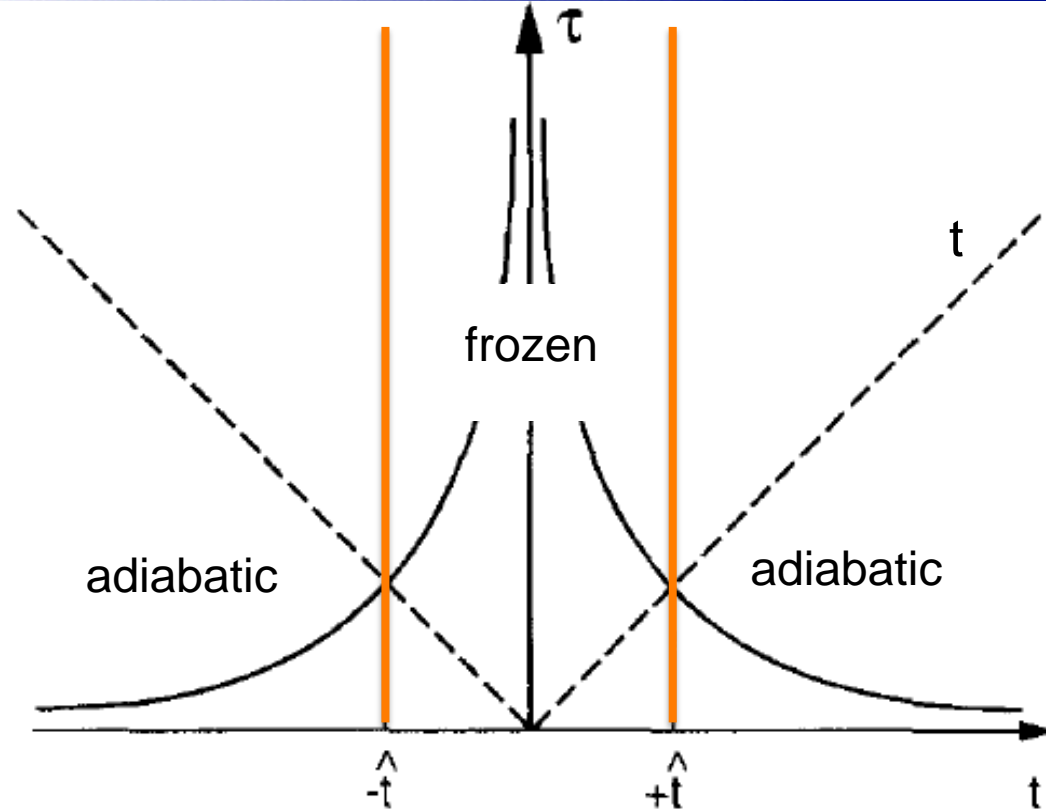


$$\varepsilon(t) = t/\tau_Q$$

$$\tau(t) = \frac{\tau_0}{|\varepsilon(t)|^{z\nu}}$$

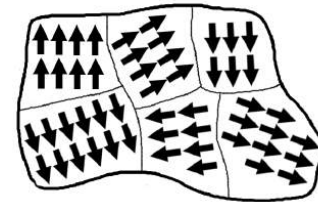


# The Kibble-Zurek mechanism



$$\varepsilon(t) = t/\tau_Q$$

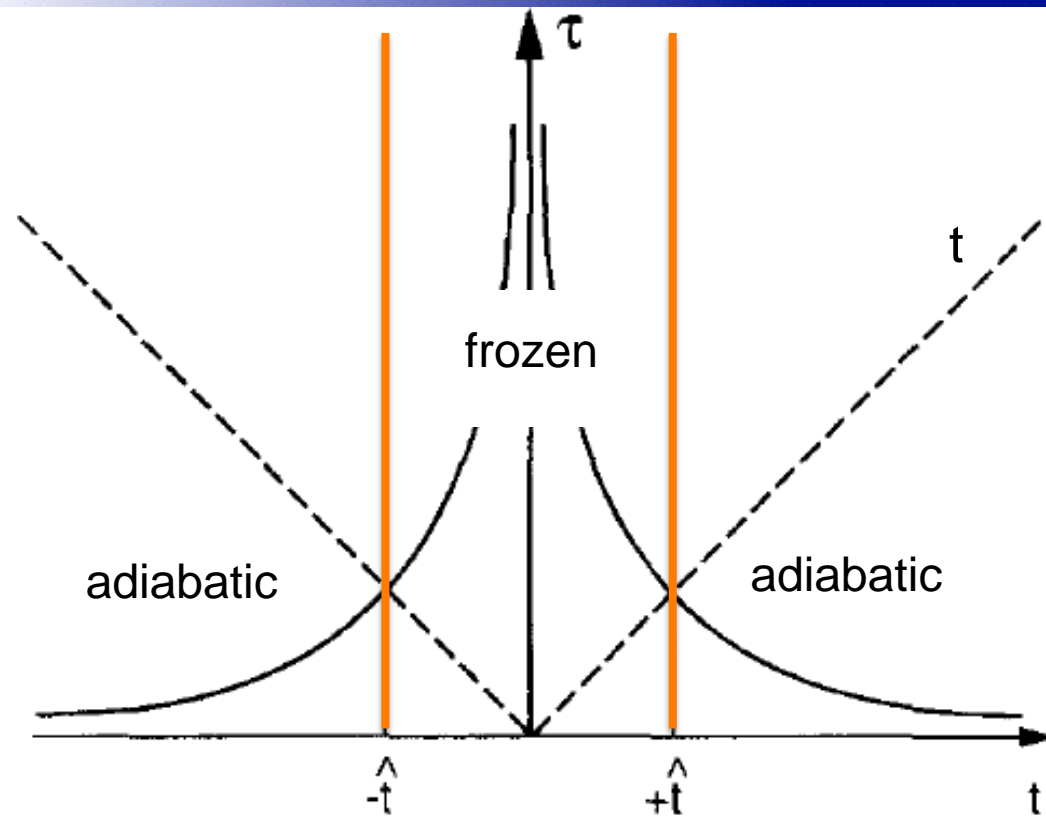
$$\tau(t) = \frac{\tau_0}{|\varepsilon(t)|^{z\nu}}$$



Domain size set by equilibrium correlation length at freeze-out time

$$\xi(t) = \frac{\xi_0}{|\varepsilon(t)|^\nu} \quad \longrightarrow \quad \hat{\xi} = \xi(\hat{t}) = \xi_0 \left( \frac{\tau_Q}{\tau_0} \right)^{\frac{\nu}{1+z\nu}}$$

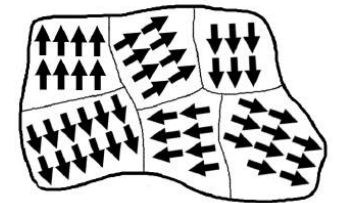
# The Kibble-Zurek mechanism



## Linear quench

$$\epsilon(t) = t/\tau_Q$$

$$\tau(t) = \frac{\tau_0}{|\epsilon(t)|^{z\nu}}$$



Density of excitations (quantum & classical)

$$n_0 \sim \xi(\hat{t}) \propto \tau_Q^{-\nu/(1+z\nu)}$$

# Universality: The Kibble-Zurek mechanism

The KZM is broadly applicable

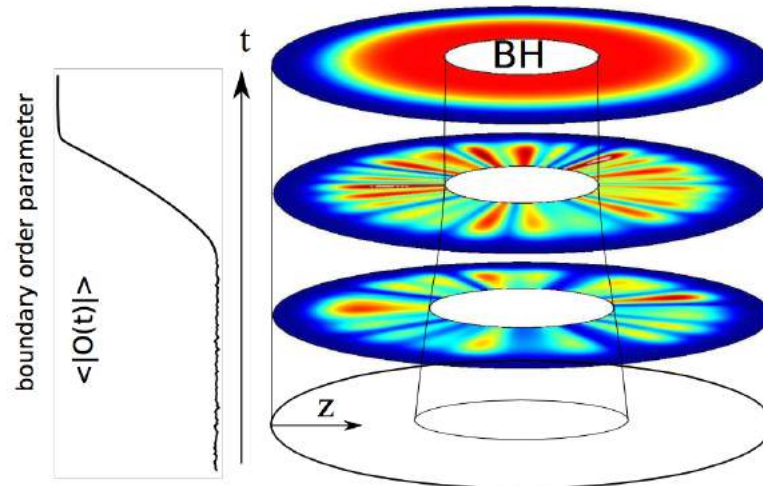
Tested numerically in integrable and nonintegrable models

Explored in many experiments, consistent with KZM

# Universality

KZM holds even in strongly coupled systems

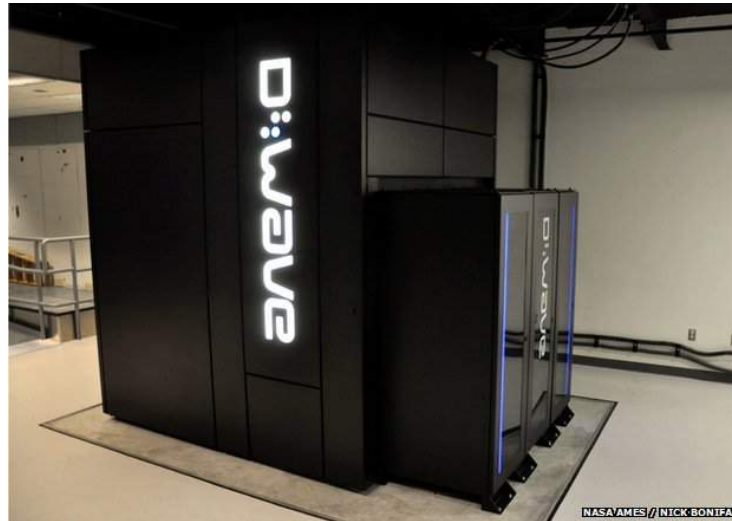
e.g. AdS-CFT/Holographic systems with no quasiparticles



J. Sonner, A. del Campo, W. H. Zurek, Nat. Commun. 6, 7406 (2015)

P. M. Chesler, A. M. Garcia-Garcia, H. Liu, PRX 5, 021015 (2015)

# KZM in AQC

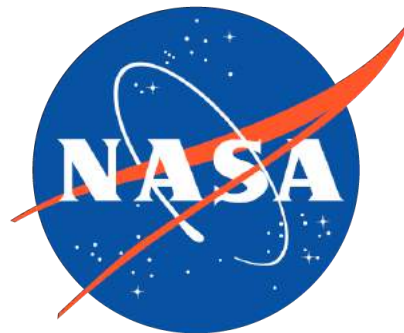
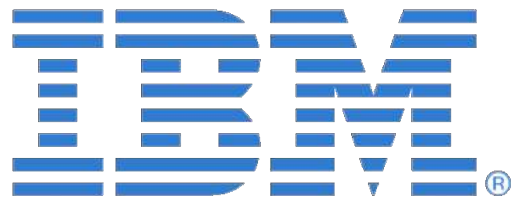
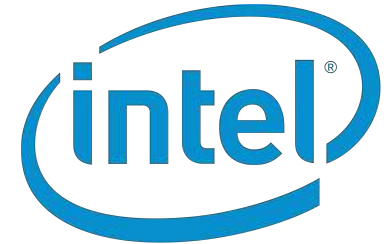


A. Dutta, A. Rahmani, A. del Campo, arXiv:1605.01062 (to appear in PRL)

M. M. Rams, M. Mohseni, A. del Campo, arXiv:1606.07740



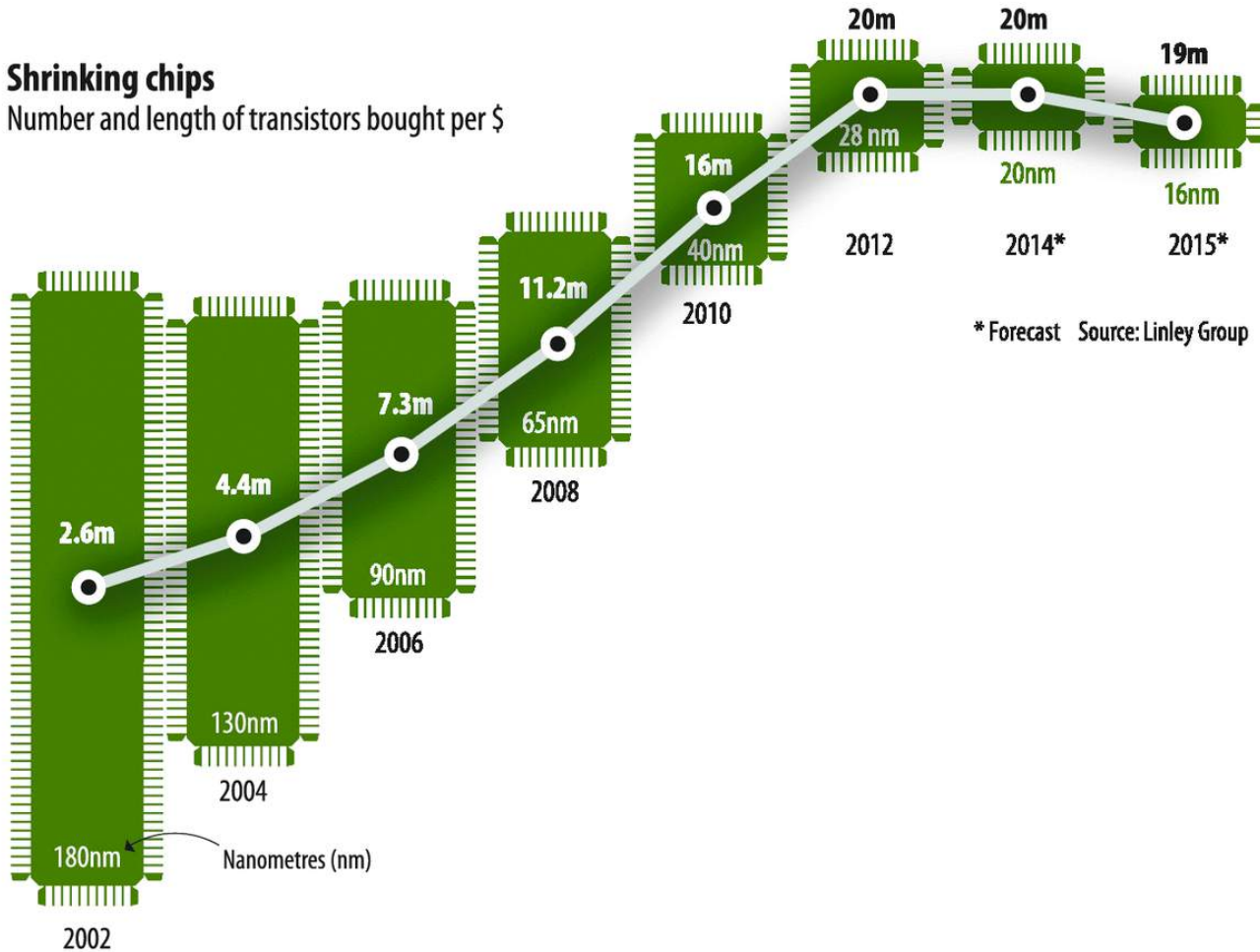
# IT companies & security agencies exploring Quantum



# Why?

## Shrinking chips

Number and length of transistors bought per \$



\* Forecast Source: Linley Group

# Why?

## MIT Technology Review

## Chip Makers Admit Transistors Are About to Stop Shrinking

In the next five years, it will be too expensive to further miniaturize—but chip makers will innovate in different ways.

by Jamie Condliffe July 25, 2016

### [International Technology Roadmap for Semiconductors Examines Next 15 Years of Chip Innovation](#)

FINAL INSTALLMENT OF BIENNIAL REPORT OUTLINES SHORT-TERM AND LONG-TERM CHALLENGES AND OPPORTUNITIES FACING SEMICONDUCTOR TECHNOLOGY

Published Friday, July 8, 2016 2:00 pm

by [Dan Rosso](#)

WASHINGTON—July 8, 2016—The Semiconductor Industry Association (SIA), representing U.S. leadership in semiconductor manufacturing, design, and research, today announced the release of the 2015 [International Technology Roadmap for Semiconductors \(ITRS\)](#), a collaborative report that surveys the technological challenges



# Adiabatic Quantum Computation?

Recast optimization problem as an annealing problem

Evolve adiabatically from a trivial Hamiltonian to a different one whose ground state encodes the solution to the optimization problem

Go Quantum (Nishimori, Farhi, ...)

PHYSICAL REVIEW E

VOLUME 58, NUMBER 5

NOVEMBER 1998

**Quantum annealing in the transverse Ising model**

Tadashi Kadowaki and Hidetoshi Nishimori

*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan*

**A Quantum Adiabatic Evolution  
Algorithm Applied to Random  
Instances of an NP-Complete  
Problem**

Edward Farhi,<sup>1\*</sup> Jeffrey Goldstone,<sup>1</sup> Sam Gutmann,<sup>2</sup>  
Joshua Lapan,<sup>3</sup> Andrew Lundgren,<sup>3</sup> Daniel Preda<sup>3</sup>



# Kibble-Zurek scaling in AQC

Density of excitations

Crossing of a quantum critical point  $n_0 \propto \tau^{-\frac{d\nu}{1+z\nu}}$

○ Quantum Ising chain  $n_0 \propto 1/\sqrt{\tau}$

○ Quantum Ising chain with quenched disorder

$$n_0 \propto 1/(\log \tau)^2$$

# Idea I

Defect suppression:  
Inhomogeneous driving

# Idea

## Phase transition induced by crossing the critical point locally

Choice of the broken symmetry biased by the neighboring regions  
that have already entered the new phase

IOP PUBLISHING  
J. Phys.: Condens. Matter 25 (2013) 404210 (10pp)

JOURNAL OF PHYSICS: CONDENSED MATTER  
doi:10.1088/0953-8984/25/40/404210

### Causality and non-equilibrium second-order phase transitions in inhomogeneous systems

A del Campo<sup>1,2</sup>, T W B Kibble<sup>3</sup> and W H Zurek<sup>1</sup>

<sup>1</sup>Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

<sup>2</sup>Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

<sup>3</sup>Blackett Laboratory, Imperial College, London SW7 2AZ, UK

International Journal of Modern Physics A  
Vol. 29, No. 8 (2014) 1430018 (49 pages)  
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DOI: 10.1142/S0217751X1430018X

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Universality of phase transition dynamics:  
Topological defects from symmetry breaking\*

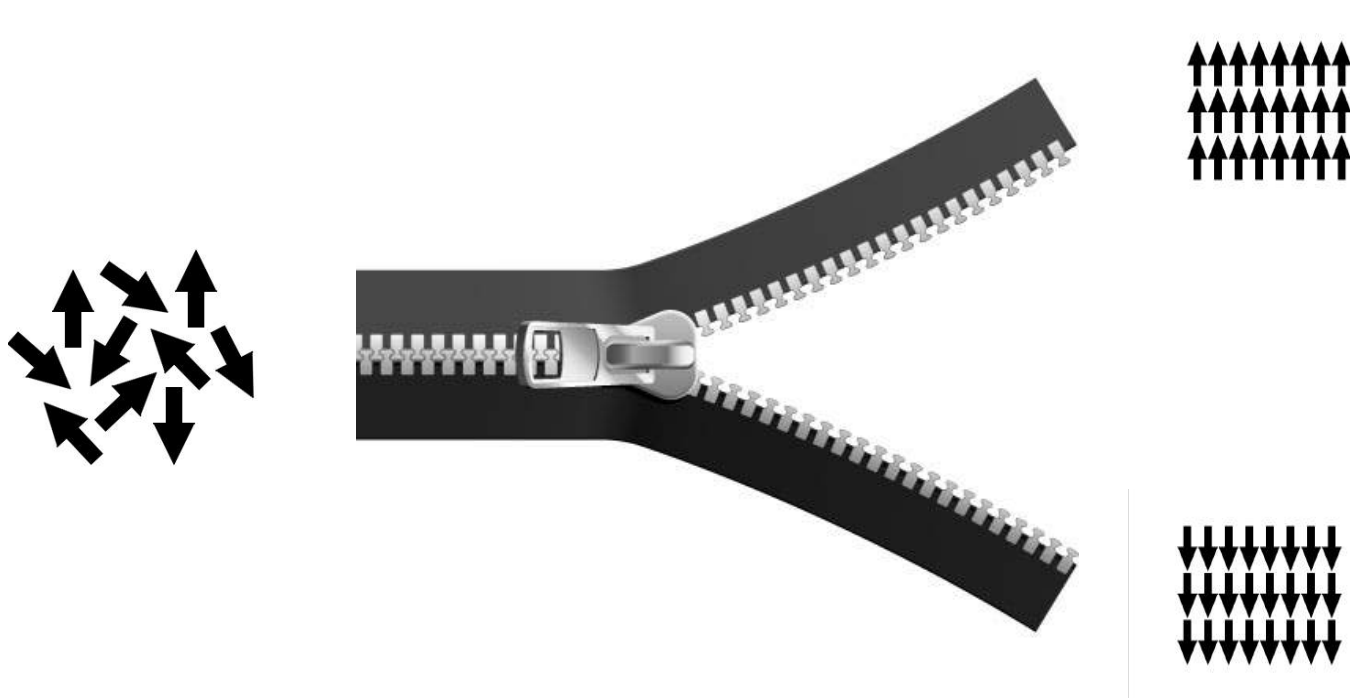
Adolfo del Campo<sup>†,‡</sup> and Wojciech H. Zurek<sup>†</sup>

<sup>†</sup>Theoretical Division, Los Alamos National Laboratory, USA

<sup>‡</sup>Center for Nonlinear Studies, Los Alamos National Laboratory, USA

# Idea

Phase transition induced by crossing the critical point locally





# Example: Structural phases in trapped ions

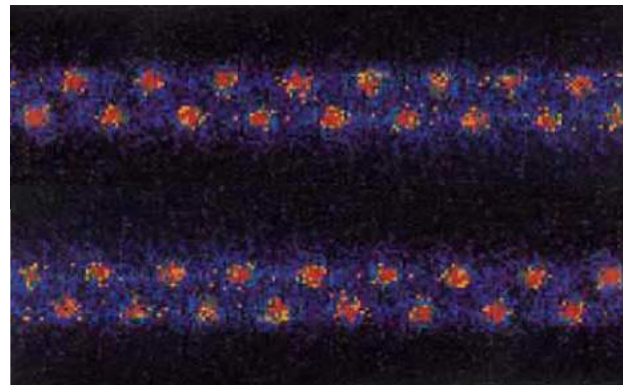
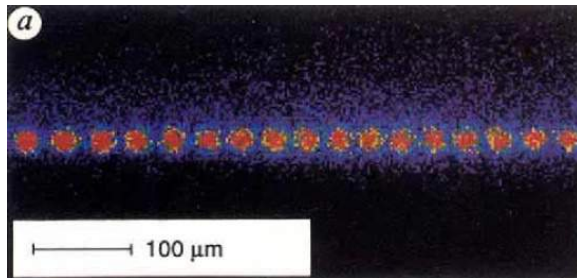
$N$  ions on a ring trap with harmonic transverse confinement

$$H = \frac{1}{2}m \sum_n \dot{\mathbf{r}}_n^2 + \frac{1}{2}m \sum_n (\nu_t^2 z_n^2) + \frac{Q^2}{2} \sum_{n \neq n'} \frac{1}{|\mathbf{r}_n - \mathbf{r}'_n|}$$

Critical transverse frequency

Linear chain

Degenerated zig-zag chains



$$\nu_t^{(c)2} = 4 \frac{Q^2}{ma(0)^3}$$

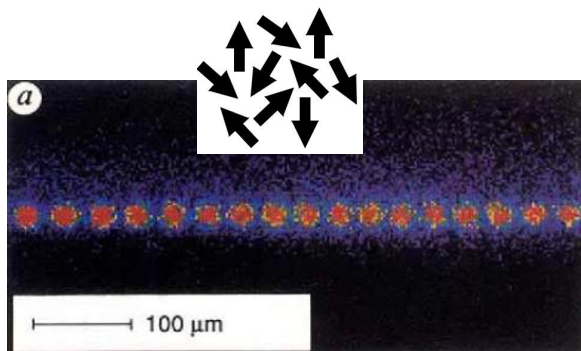
# Structural phases in trapped ions

$N$  ions on a ring trap with harmonic transverse confinement

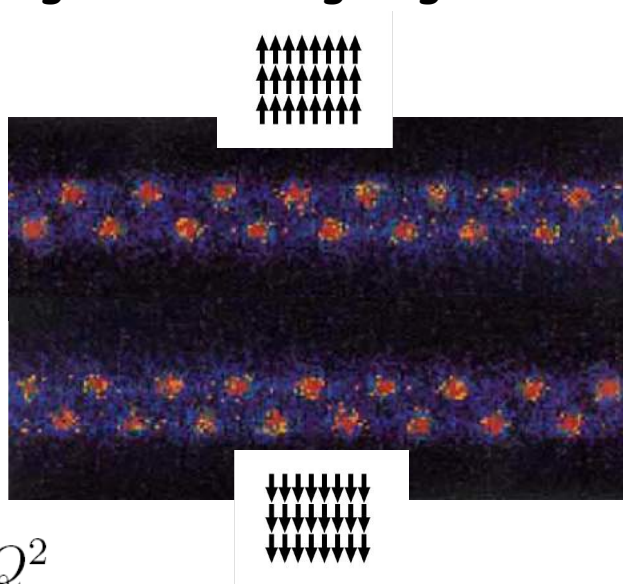
$$H = \frac{1}{2}m \sum_n \dot{\mathbf{r}}_n^2 + \frac{1}{2}m \sum_n (\nu_t^2 z_n^2) + \frac{Q^2}{2} \sum_{n \neq n'} \frac{1}{|\mathbf{r}_n - \mathbf{r}'_n|}$$

Critical transverse frequency

Linear chain



Degenerated zig-zag chains

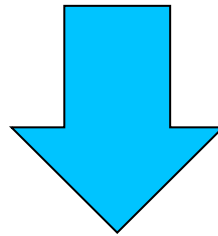
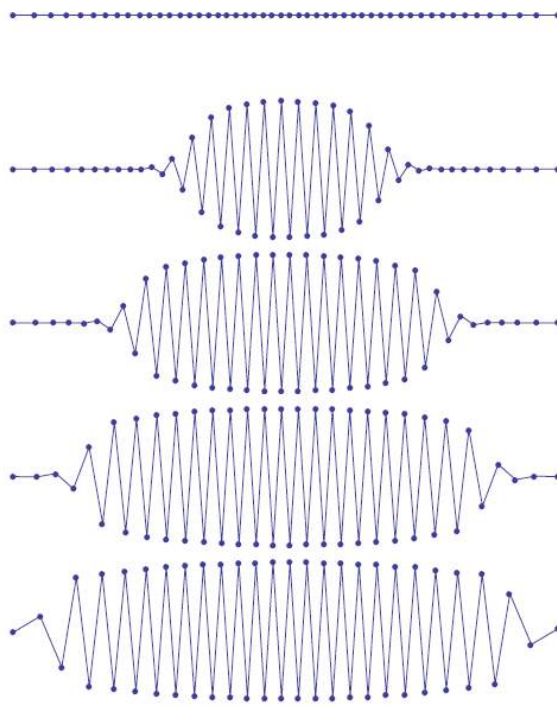


$$\nu_t^{(c)2} = 4 \frac{Q^2}{ma(0)^3}$$

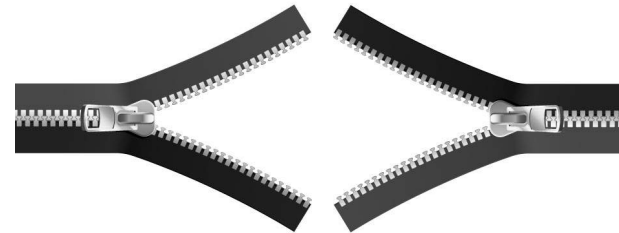
# Inhomogeneous driving

Axial and transverse harmonic potential (instead of a ring trap)

$$H = \frac{1}{2}m \sum_n \dot{\mathbf{r}}_n^2 + \frac{1}{2}m \sum_n (\nu_t^2 z_n^2 + \nu^2 x_n^2) + \frac{Q^2}{2} \sum_{n \neq n'} \frac{1}{|\mathbf{r}_n - \mathbf{r}'_n|}$$



Spatially dependent critical frequency  
(within LDA)

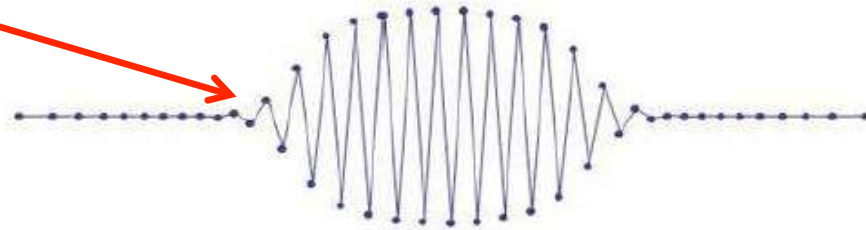


# Inhomogeneous driving

Causality reduces the effective system for defect formation

Front velocity  $v_F$

Sound velocity  $c$



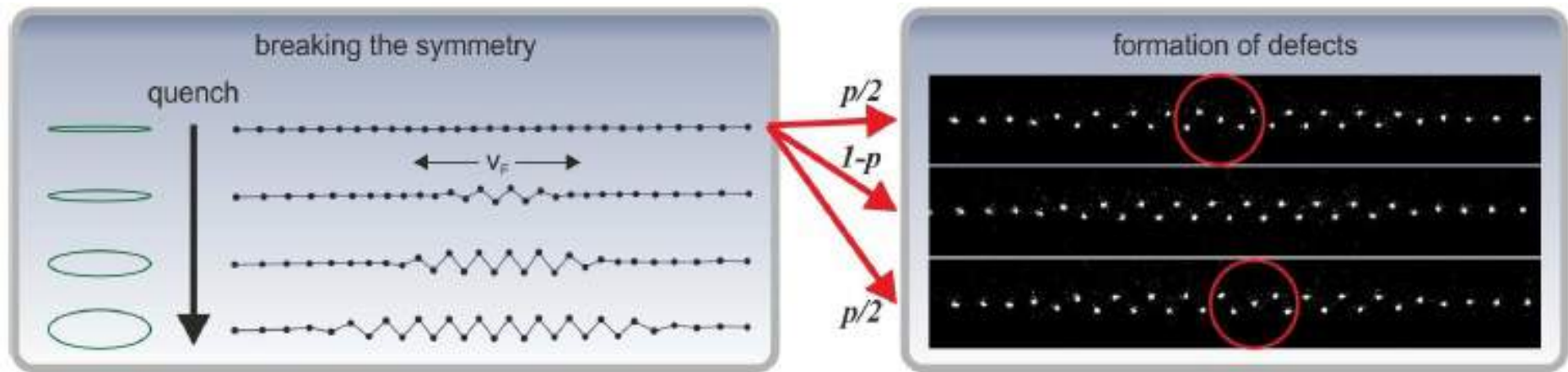
Adiabatic dynamics  $v_F < c$

Kink formation  $v_F > c$  in an effective system size  $L_{\text{eff}}(\tau_Q)$

Density of defects: New power law 
$$d = \frac{L_{\text{eff}}(\tau_Q)}{\hat{\xi}} \frac{1}{L} \sim \left( \frac{1}{\tau_Q} \right)^{4/3}$$

# In the lab

Collaboration with  
Mehlstaubler's group at PTB

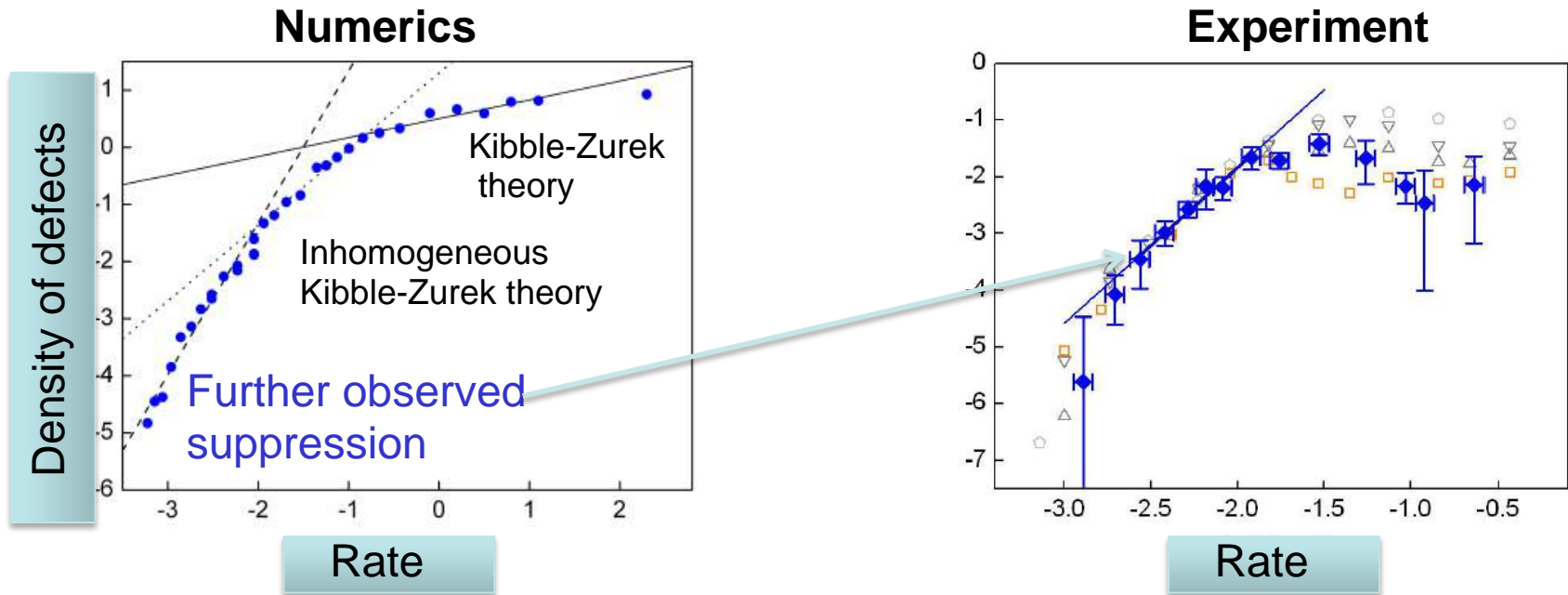


32 ions, only  $\{0,1\}$  defects per realization



Kibble-Zurek theory fails at the onset of adiabatic dynamics

Regime of interest to quantum simulation

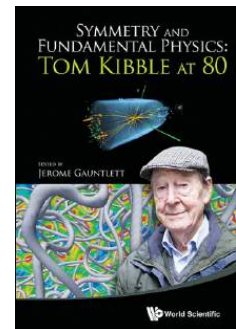


32 ions, only  $\{0,1\}$  defects per realization  
Pyka et al. Nat. Commun. 4, 2291 (2013)

# Several experiments

$$n \propto \tau_Q^{-\alpha}.$$

Group	Number of ions	Kink number	Fitted exponent $\alpha$
Mainz University <sup>14</sup>	16	{0, 1}	$2.68 \pm 0.06$
PTB <sup>15</sup>	$29 \pm 2$	{0, 1}	$2.7 \pm 0.3$
Simon Fraser University <sup>13</sup>	$42 \pm 1$	{0, 2}	$3.3 \pm 0.2$



# Inhomogeneous driving

Inhomogeneous driving helps to suppress defect formation

Applicability to adiabatic quantum computation?

It does NOT require diagonalization of the Hamiltonian

Experimental tests restricted to small systems

Onset of adiabatic dynamics needs theory

IOP PUBLISHING

J. Phys.: Condens. Matter 25 (2013) 404210 (10pp)

JOURNAL OF PHYSICS: CONDENSED MATTER

doi:10.1088/0953-8984/25/40/404210

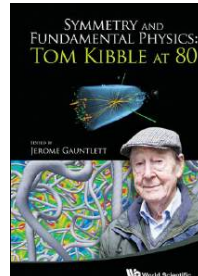
## Causality and non-equilibrium second-order phase transitions in inhomogeneous systems

A del Campo<sup>1,2</sup>, T W B Kibble<sup>3</sup> and W H Zurek<sup>1</sup>

<sup>1</sup> Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

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<sup>3</sup> Blackett Laboratory, Imperial College, London SW7 2AZ, UK



AdC, W. H. Zurek  
Int. J. Mod. Phys. A 29, 1430018 (2014)



# The Kibble-Zurek scalings in AQC

Density of excitations

Crossing of a quantum critical point  $n_0 \propto \tau^{-\frac{d\nu}{1+z\nu}}$

○ Quantum Ising chain  $n_0 \propto 1/\sqrt{\tau}$

○ Quantum Ising chain with quenched disorder

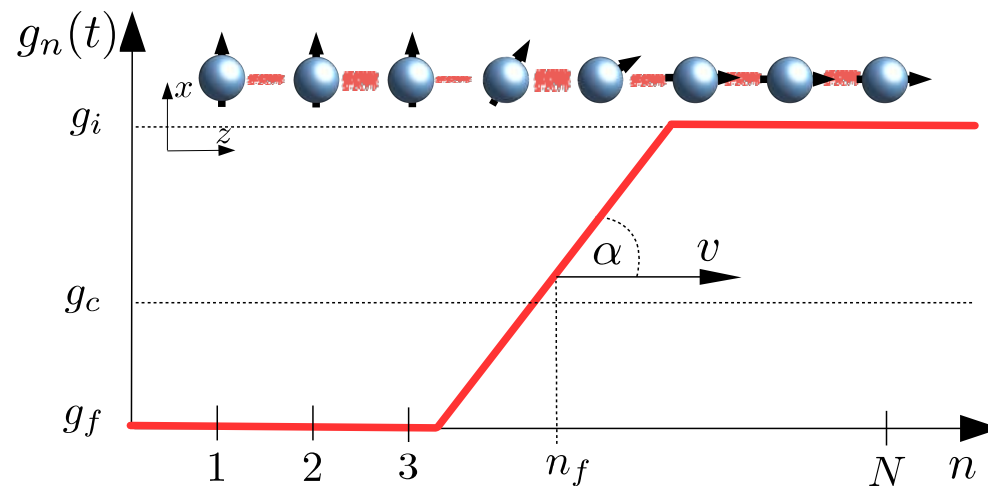
$$n_0 \propto 1/(\log \tau)^2$$

# Inhomogeneous AQC

Transverse-field quantum Ising chain with (weak) disorder

$$\hat{H}/\mathcal{J} = - \sum_{n=1}^{N-1} J_n \sigma_n^x \sigma_{n+1}^x - \sum_{n=1}^N g_n \sigma_n^z$$

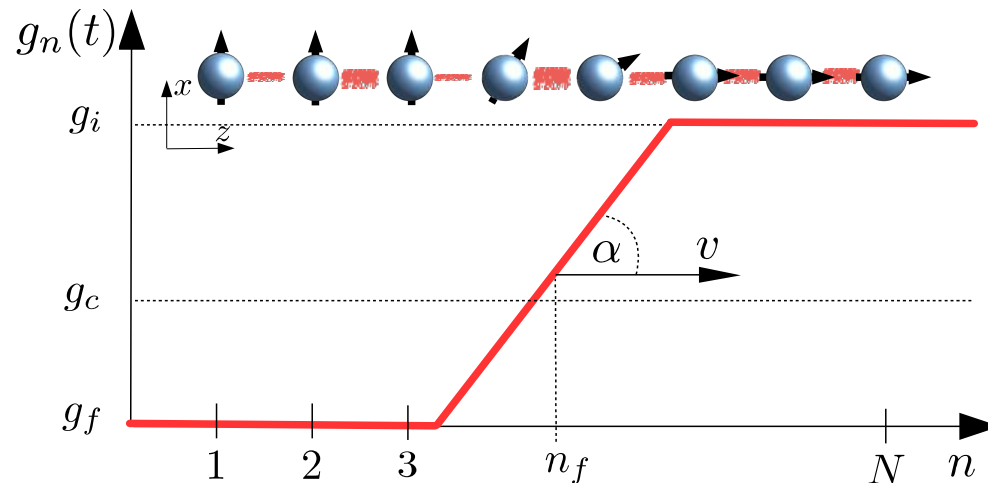
$$P(J_n) = \begin{cases} 1 & \text{for } J_n \in (1/2, 3/2), \\ 0 & \text{otherwise.} \end{cases}$$



# Inhomogeneous KZM

Transverse-field quantum Ising chain with weak disorder

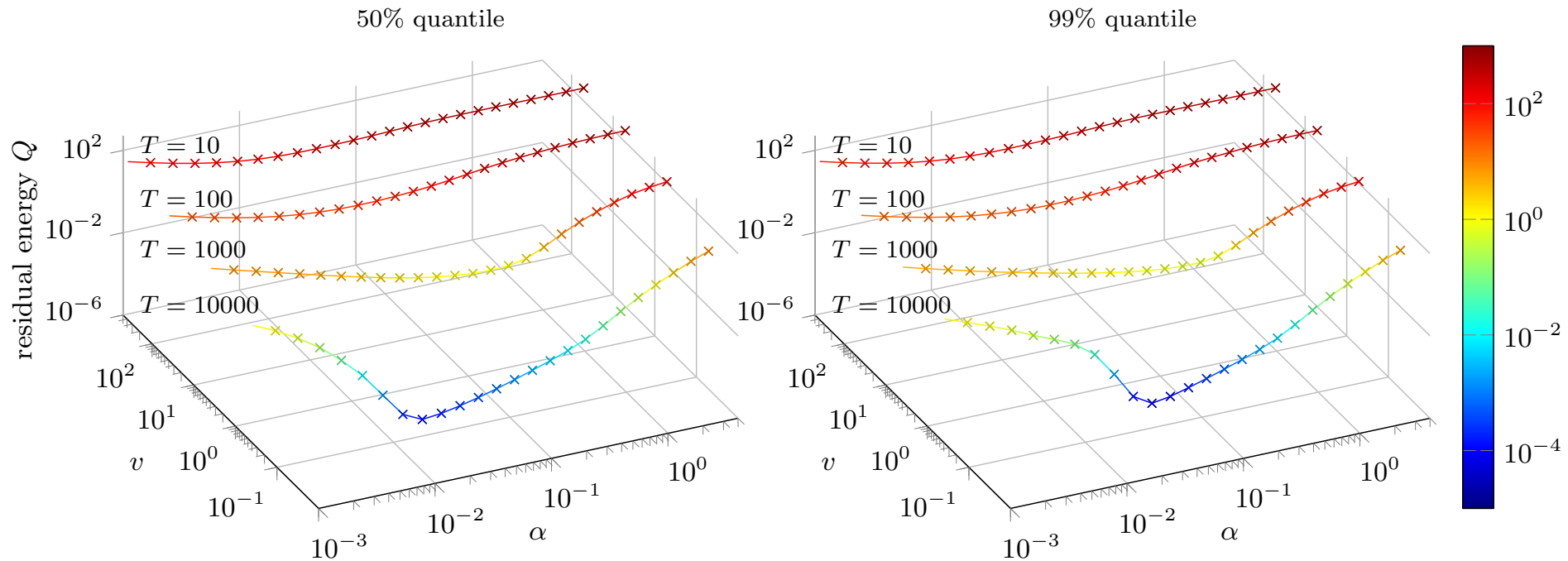
$$\hat{\xi} \sim \alpha^{-\frac{\nu}{\nu+1}} \quad v_t \sim \alpha^{\frac{(z-1)\nu}{\nu+1}}$$



Naively setting  $z \rightarrow \infty$  suggests Inhomogeneous driving does not help

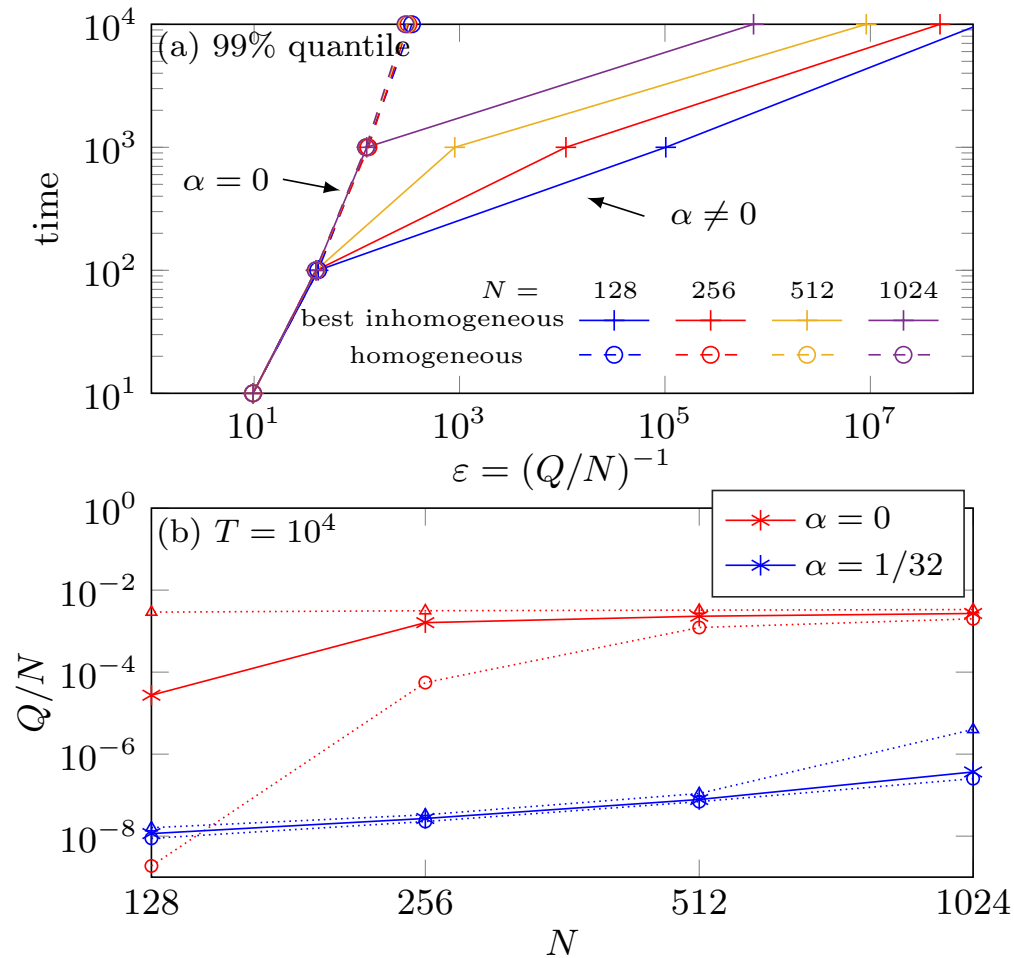
# Inhomogeneous AQC

## Performance



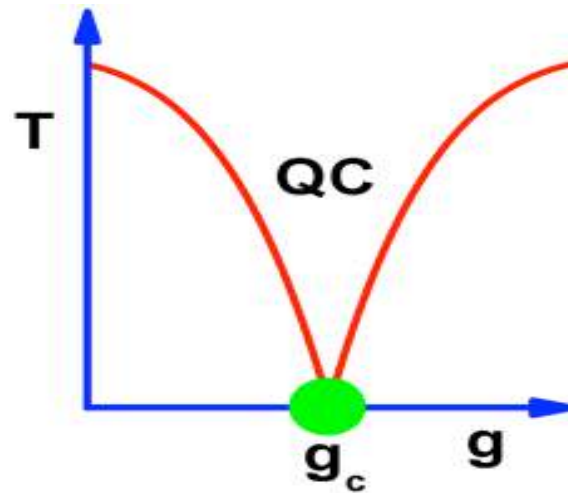
# Inhomogeneous AQC

Supremacy over conventional homogeneous protocols



# Idea II

## Counterdiabatic driving in a Quantum Phase Transition



# Example: 1d Quantum Ising Chain

Ising chain Hamiltonian  $\hat{H}_0(t) = - \sum_{n=1}^N [\sigma_n^x \sigma_{n+1}^x + g(t) \sigma_n^z]$

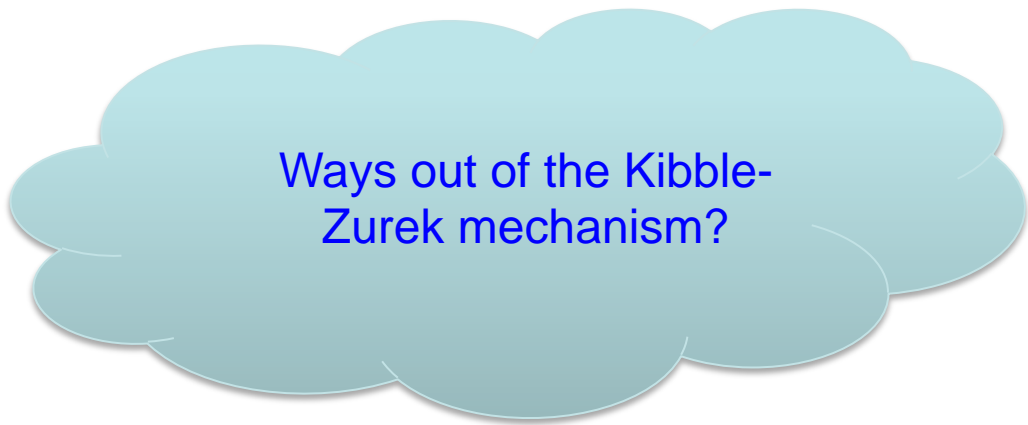
Critical point  $g_c = 1$

$g \gg 1$   $|\rightarrow\rightarrow\rightarrow \dots \rightarrow\rangle$   
z-axis

$g \ll 1$   $|\uparrow\uparrow\uparrow \dots \uparrow\rangle$   
 $|\downarrow\downarrow\downarrow \dots \downarrow\rangle$

x-axis

Excitations:  $|\dots \uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow \dots\rangle$



# Counterdiabatic driving: Ising Chain

Counterdiabatic driving?



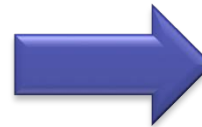
Auxiliary control

$$\hat{H}_1(t) = i\hbar \sum_n (|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle|n\rangle\langle n|)$$

**Diagonalization: Jordan Wigner transformation + Fourier transform**

$$\hat{H}_0(t) = 2 \sum_{k>0} \Psi_k^\dagger [\sigma_k^z (g(t) - \cos k) + \sigma_k^x \sin k] \Psi_k$$

$$\hat{H}_1(t) = -\dot{g}(t) \sum_{k>0} \frac{1}{2} \frac{\sin k}{g^2 + 1 - 2g \cos k} \Psi_k^\dagger \sigma_k^y \Psi_k$$



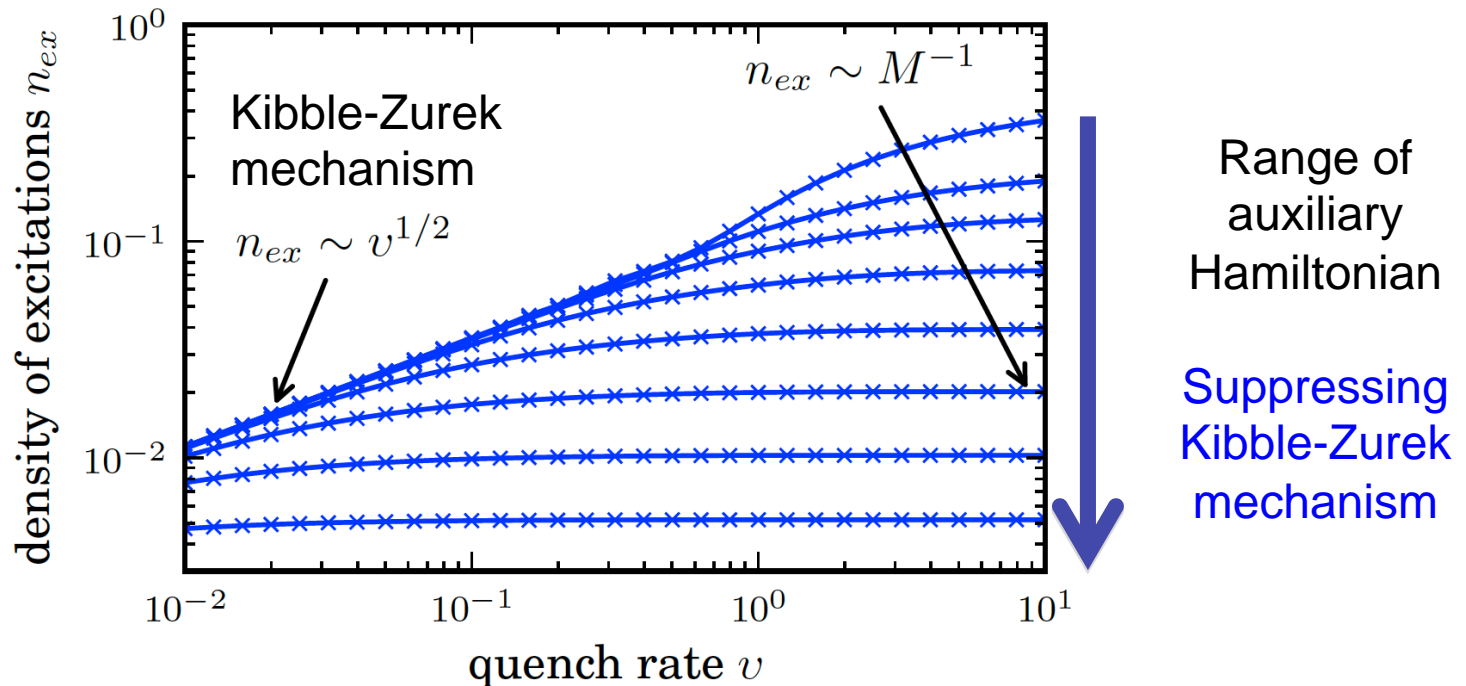
Long-range many-body interaction!



# Truncated Auxiliary Hamiltonian

Quench through the critical point  $g(t) = g_c - vt$

Truncated Auxiliary Hamiltonian

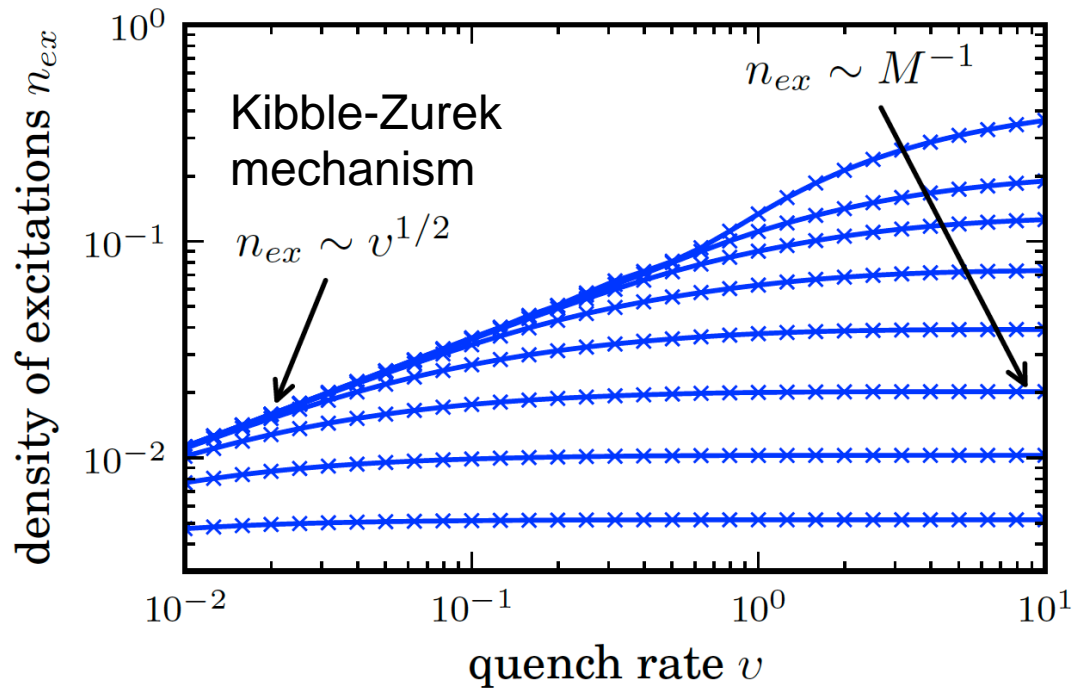


# Truncated Auxiliary Hamiltonian

Quench through the critical point  $g(t) = g_c - vt$

Truncated Auxiliary Hamiltonian

Experimentalist's view



# Tailoring control fields



H. Saberi, T. Opatrný, K. Mølmer, AdC, Phys. Rev. A 90, 060301(R) (2014)

# Tailoring auxiliary interactions

Set of available controls  $\{L_k\}$

Approximated Auxiliary Hamiltonian  $\tilde{H}_1 = \sum_{k=1}^K \alpha_k L_k$

Minimize the norm  $\min_{\{\alpha_k\}} \|(H_1 - \tilde{H}_1)|GS(t)\rangle\|^2$

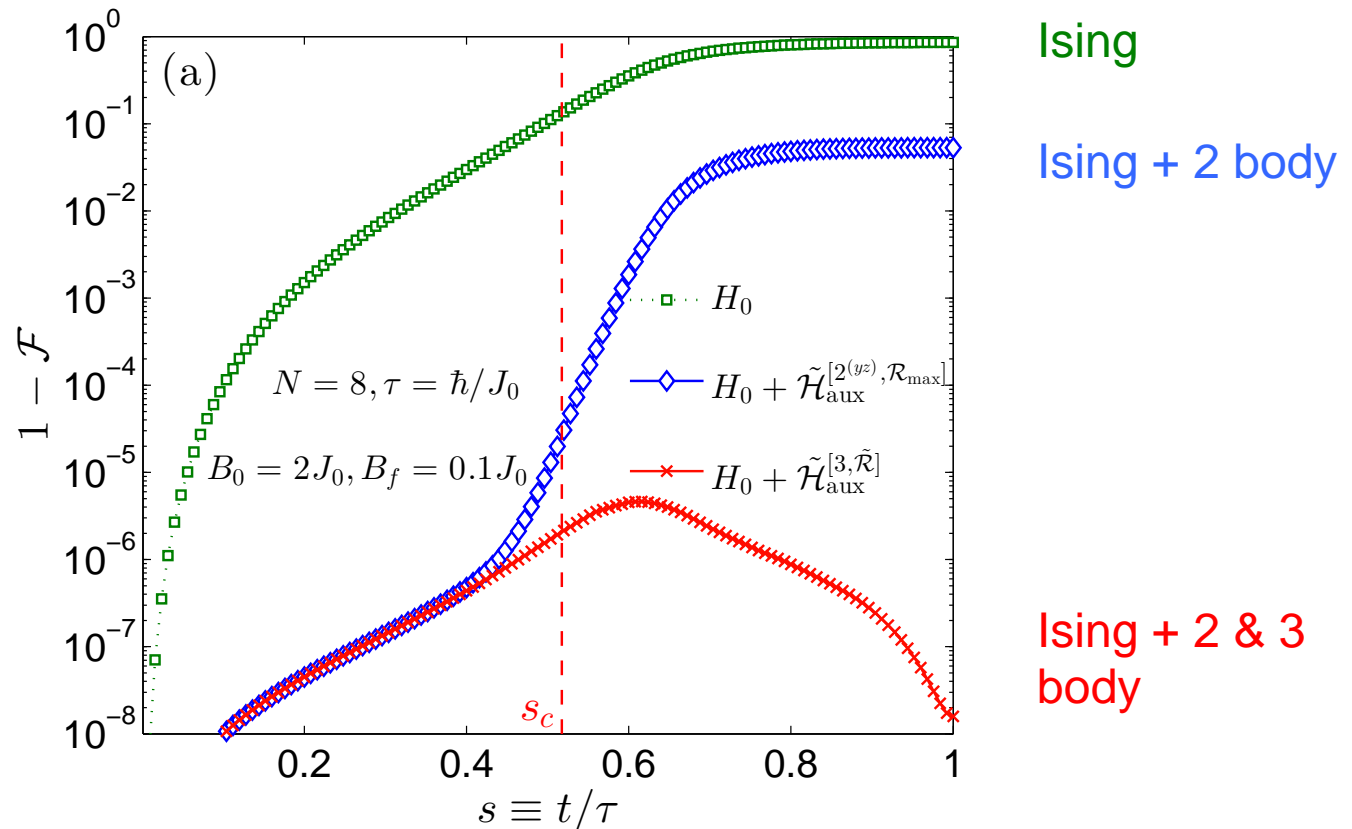


T. Opatrny, K. Mølmer, NJP 16, 015025 (2014)

H. Saberi, T. Opatrny, K. Mølmer, AdC, Phys. Rev. A 90, 060301(R) (2014)

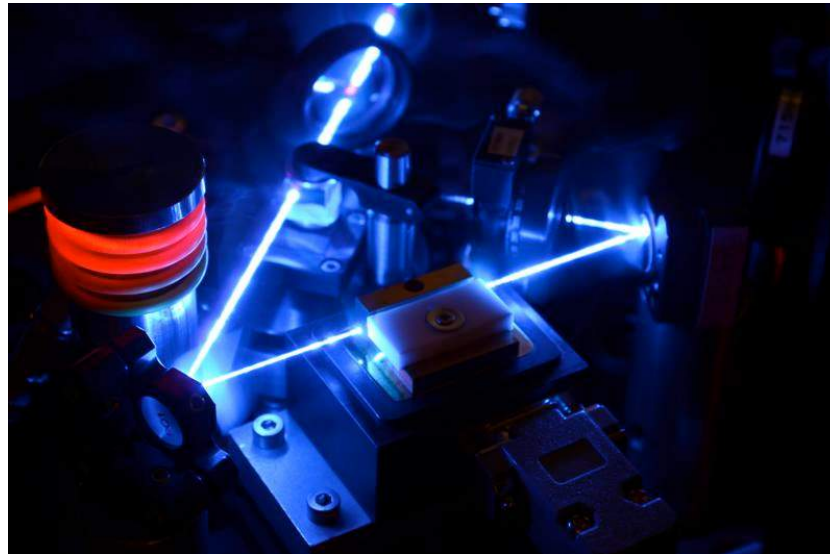
# Suppressing KZM/excitations

Approximated Auxiliary Hamiltonian 
$$\tilde{H}_1(t) = \sum_{i_1, i_2} h_{i_1, i_2}^{y, z}(t) \sigma_{i_1}^y \otimes \sigma_{i_2}^z$$



H. Saberi, T. Opatrny, K. Mølmer, AdC, Phys. Rev. A 90, 060301(R) (2014)

# Testing KZM in a Quantum Simulator



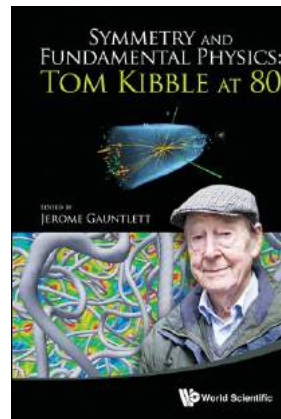
Jin-Ming Cui, Yun-Feng Huang, Zhao Wang, Dong-Yang Cao, Jian Wang, Wei-Min Lv,  
Yong Lu, Le Luo, Adolfo del Campo, Yong-Jian Han, Chuan-Feng Li, Guang-Can Guo,

arXiv:1505.05734

# Case for Quantum Simulation

- Experimental tests of KZM focused on **thermal** phase transitions
- Challenges: varying quench rate, counting of defects, determining universality class

A. del Campo, W. H. Zurek  
Int. J. Mod. Phys. A **29**, 1430018 (2014)



- Experiments in the **Quantum** regime are even more difficult
- Additional challenges: ground-state cooling, decoherence

# Quantum Simulation

- Use a simple quantum system to simulate a more complex one



## **Simulating Physics with Computers**

**Richard P. Feynman**

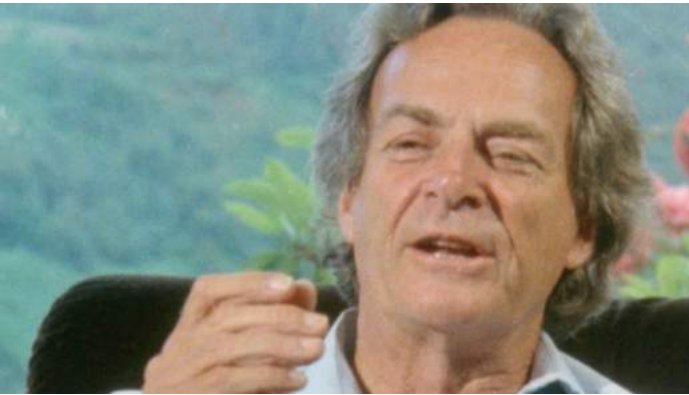
*Department of Physics, California Institute of Technology, Pasadena, California 91107*

*Received May 7, 1981*



# Quantum Simulation

- Use a simple quantum system to simulate a more complex one



## **Simulating Physics with Computers**

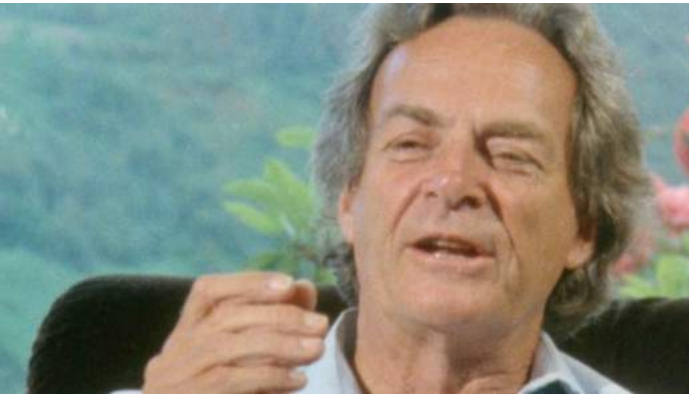
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# Quantum Simulation

- Use a simple quantum system to simulate a more complex one



## **Simulating Physics with Computers**

**Richard P. Feynman**

*Department of Physics, California Institute of Technology, Pasadena, California 91107*

*Received May 7, 1981*

- Testing KZM in the Quantum Regime
  - Simulated system: Quantum Phase Transition in 1D Ising spin chain
  - Quantum Simulator: a single qubit, trapped ion

# Simulated system: 1d Quantum Ising Chain

Ising chain Hamiltonian  $\hat{H}_0(t) = - \sum_{n=1}^N [\sigma_n^x \sigma_{n+1}^x + g(t) \sigma_n^z]$

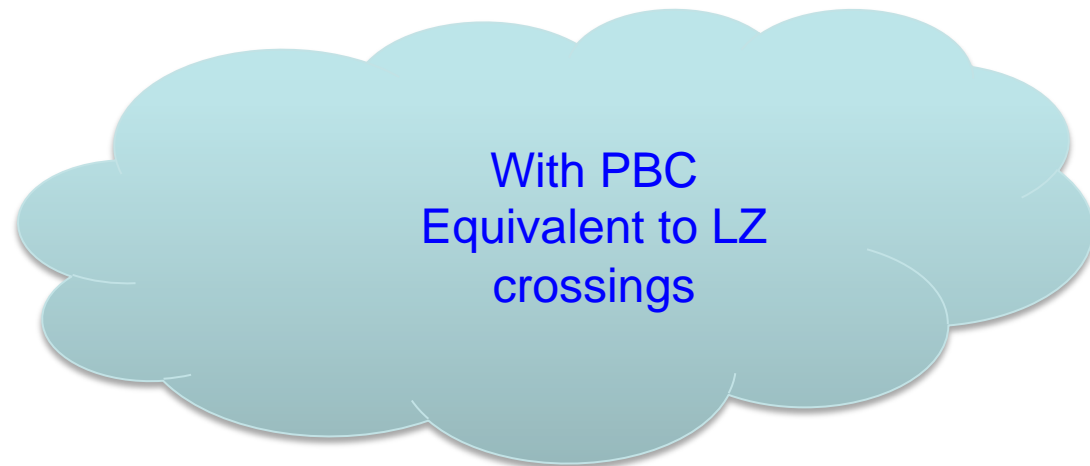
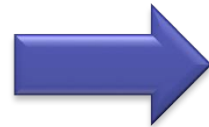
Critical point  $g_c = 1$

$g \gg 1$   $|\rightarrow\rightarrow\rightarrow \dots \rightarrow\rangle$   
z-axis

$g \ll 1$   $|\uparrow\uparrow\uparrow \dots \uparrow\rangle$   
 $|\downarrow\downarrow\downarrow \dots \downarrow\rangle$

Excitations:  $|\dots \uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow \dots\rangle$

x-axis



# Simulated system: 1d Quantum Ising Chain

Ising chain Hamiltonian  $\hat{H}_0(t) = - \sum_{n=1}^N [\sigma_n^x \sigma_{n+1}^x + g(t) \sigma_n^z]$

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$g \gg 1$   $|\rightarrow\rightarrow\rightarrow \dots \rightarrow\rangle$   $g \ll 1$   $|\uparrow\uparrow\uparrow \dots \uparrow\rangle$   
 z-axis  $|\downarrow\downarrow\downarrow \dots \downarrow\rangle$

Excitations  $|\dots \uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow \dots\rangle$  x-axis

Mapping via Jordan-Wigner & Fourier Transforms [Dziarmaga PRL '05]

$$i\hbar \frac{d}{d\tau} \begin{bmatrix} \mu_k \\ \nu_k \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\Delta_k \tau & 1 \\ 1 & \Delta_k \tau \end{bmatrix} \begin{bmatrix} \mu_k \\ \nu_k \end{bmatrix}$$

$$k = \pm \frac{1}{2} \frac{2\pi}{Na}, \dots, \pm \left(\frac{N}{2} - \frac{1}{2}\right) \frac{2\pi}{Na}$$

# Simulated system: 1d Quantum Ising Chain

Ising chain Hamiltonian  $\hat{H}_0(t) = - \sum_{n=1}^N [\sigma_n^x \sigma_{n+1}^x + g(t) \sigma_n^z]$

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$g \gg 1$   $|\rightarrow\rightarrow\rightarrow \dots \rightarrow\rangle$   $g \ll 1$   $|\uparrow\uparrow\uparrow \dots \uparrow\rangle$   
 $z$ -axis  $|\downarrow\downarrow\downarrow \dots \downarrow\rangle$

Excitations  $|\dots \uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow \dots\rangle$   $x$ -axis

Mapping via Jordan-Wigner & Fourier Transforms [Dziarmaga PRL '05]

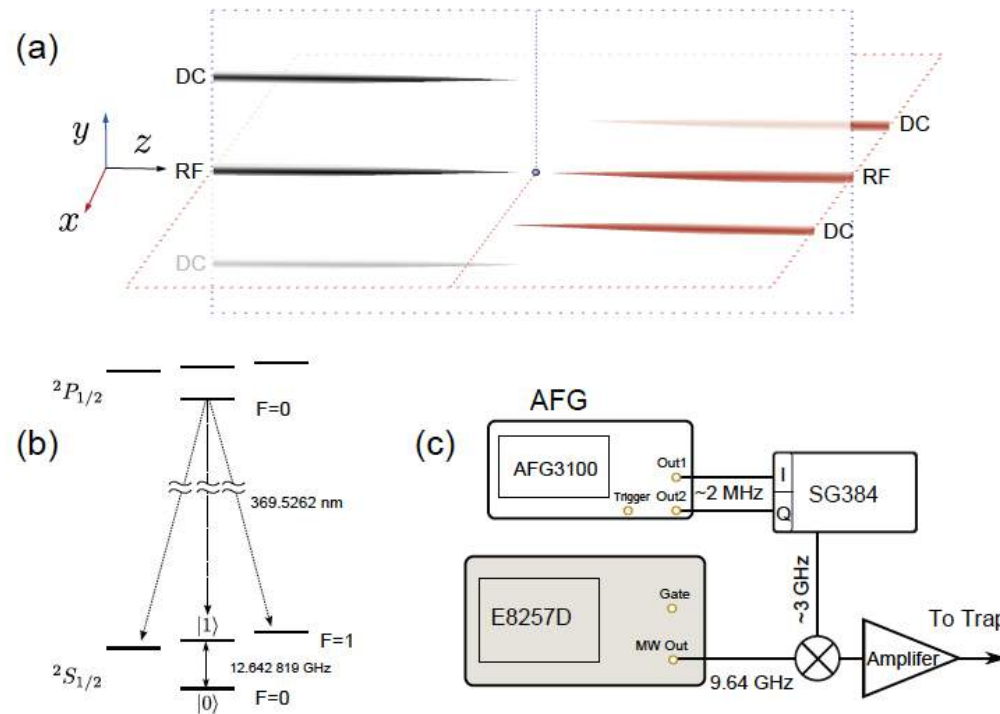
$$n_{ex} = \frac{1}{N} \sum_n \frac{1}{2} \langle 1 - \sigma_n^z \sigma_{n+1}^z \rangle = \frac{1}{N} \sum_k \gamma_k^\dagger \gamma_k$$

# Quantum Simulator: trapped ion

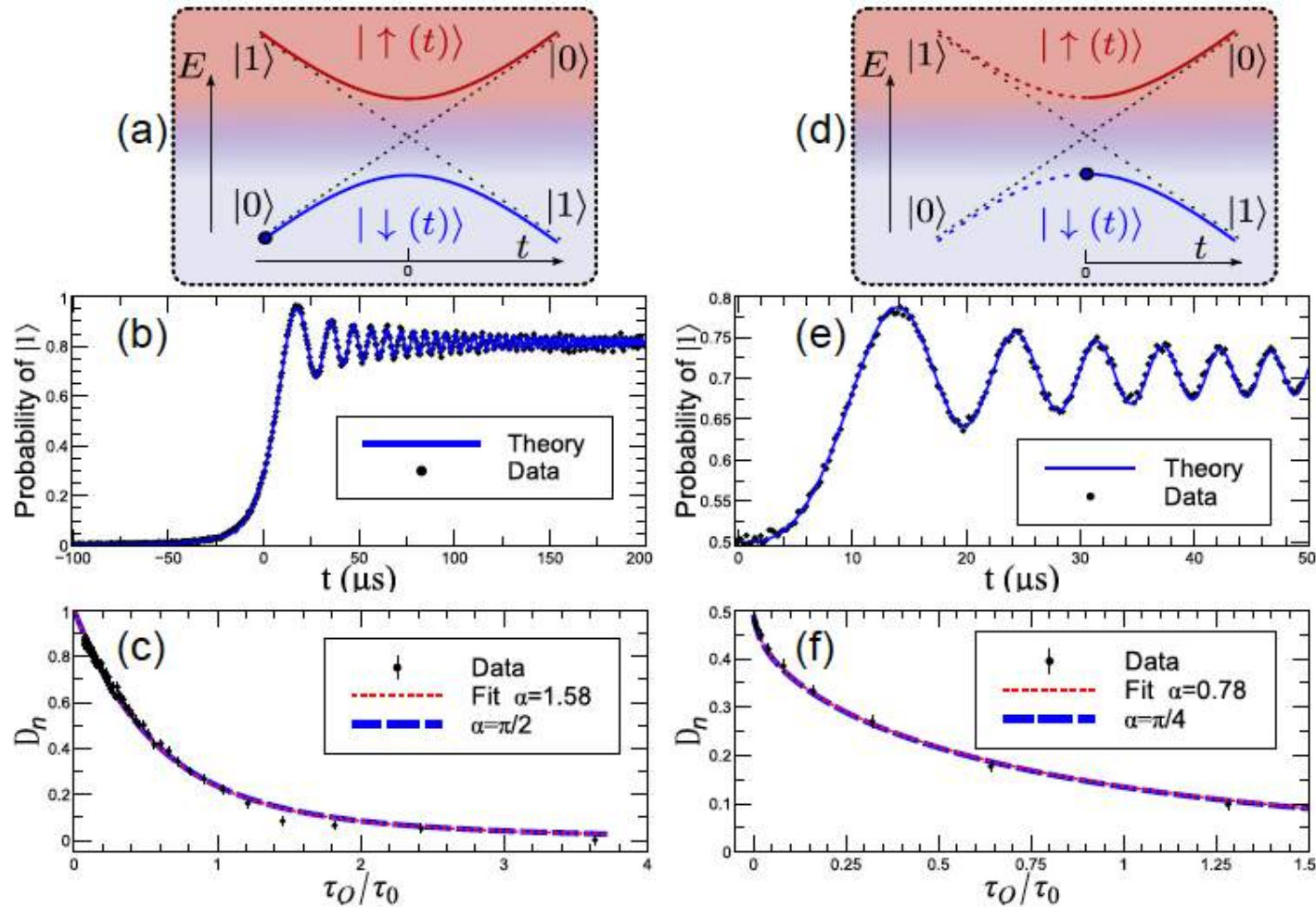
$^{171}\text{Yb}^+$  confined in a Paul trap

two hyperfine states  $|0\rangle$  &  $|1\rangle$

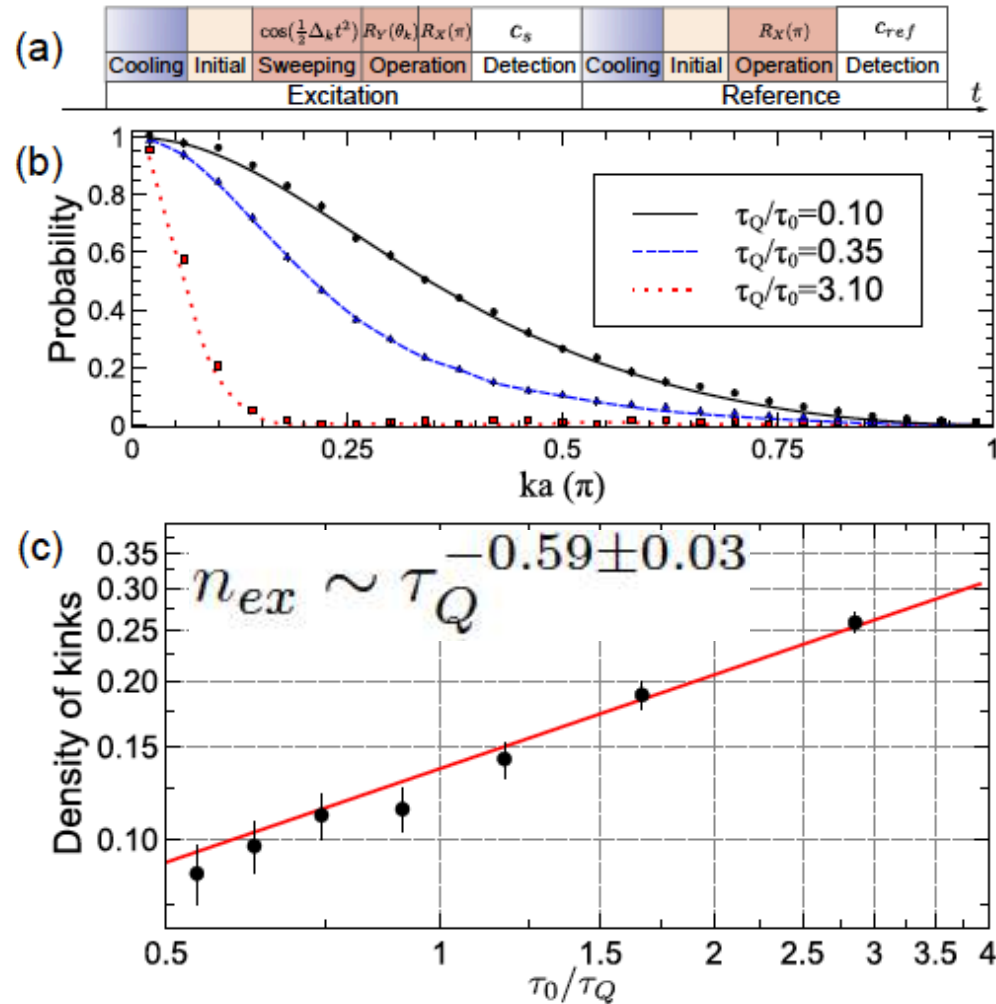
Microwave control



# Testing Landau-Zener dynamics



# Observing Quantum Kibble-Zurek scaling





# Quantum Simulation of QPT dynamics

First Experimental Demonstration of Kibble-Zurek scaling in the Quantum Regime

Ion-trap quantum simulation can be applied to

Nonlinear quenches,

Quantum control,

Shortcuts to Adiabaticity with auxiliary interactions,

Dephasing, Decoherence,

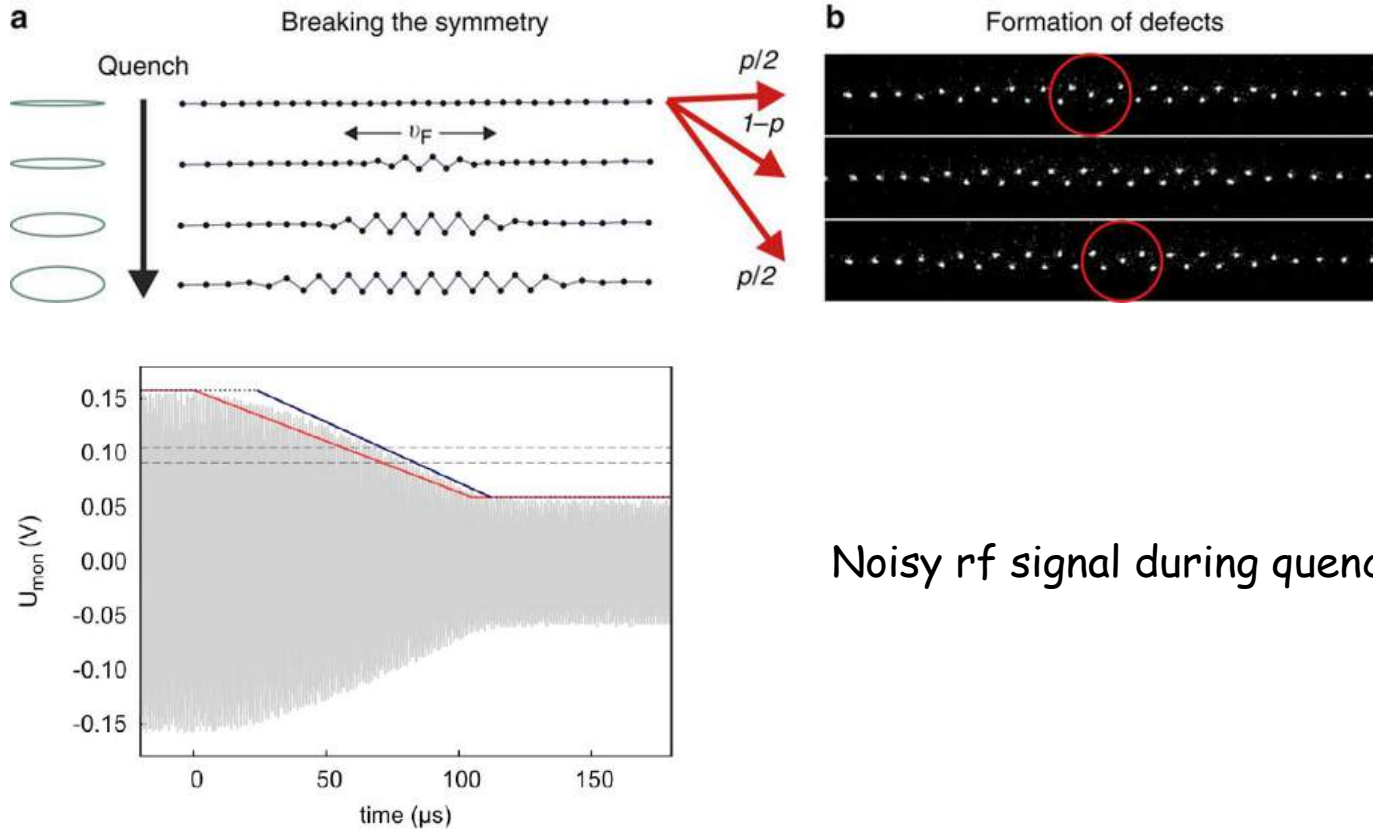
...

# Noise in the controls



# Noisy control fields

Present in all experiments, e.g., KZM test squeezing an ion chain

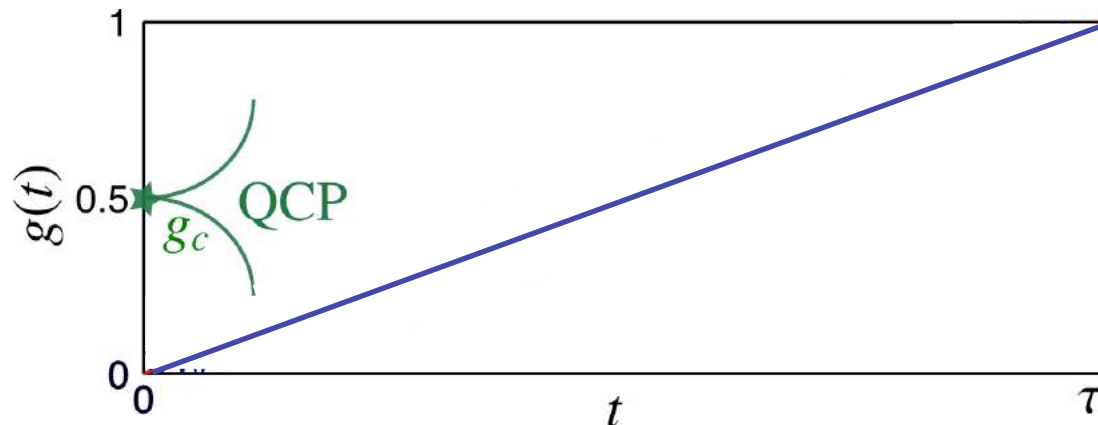


Noisy rf signal during quench

# Quantum Annealing Protocol

Example

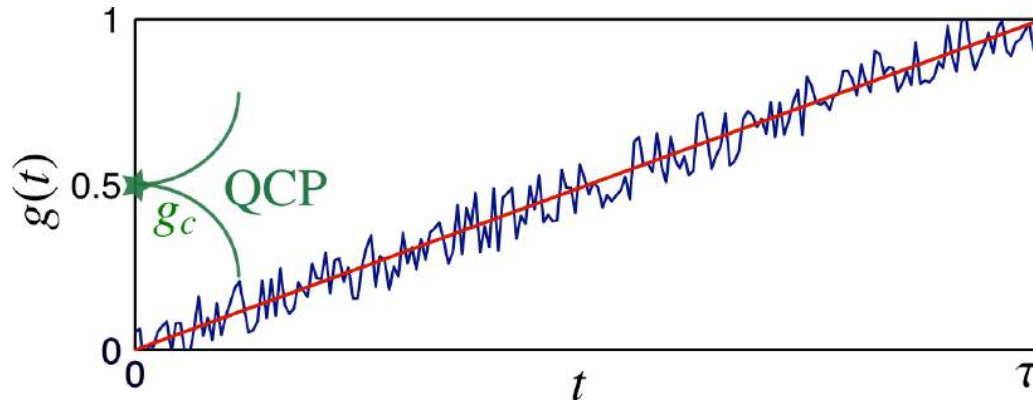
$$H = -\Lambda \sum_{n=1}^N \{ [1 - g(t)] \hat{\sigma}_n^x + g(t) \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z \}$$



$$H = -\Lambda \sum_{n=1}^N \hat{\sigma}_n^x$$

$$H = -\Lambda \sum_{n=1}^N \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z$$

# Noise in the control fields



Noisy control field

$$g(t) = t/\tau + \gamma(t), \quad 0 < t < \tau$$

$$\langle \gamma(t)\gamma(t') \rangle = W^2 \delta(t - t')$$

# Stochastic many-body Hamiltonians

Full system

$$H(t) = H_0(t) + \gamma(t)V$$

Deterministic and stochastic parts

$$H_0(t) = - \sum_{n=1}^N \{ [1 - g_0(t)] \hat{\sigma}_n^x + g_0(t) \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z \}$$

$$V = - \sum_{n=1}^N (-\hat{\sigma}_n^x + \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z)$$

Stochastic Schrodinger equation

$$i \frac{d}{dt} |\psi(t)\rangle = [H_0(t) + \gamma(t)V] |\psi(t)\rangle$$

# Noise-Averaged dynamics

Density matrix averaged over realizations



$$\rho(t) = \langle \rho_{\text{st}}(t) \rangle = \langle |\psi(t)\rangle \langle \psi(t)| \rangle$$

Master equation requires stochastic unravelling

$$\frac{d}{dt}\rho(t) = -i[H_0(t), \rho] - \int_0^t ds \langle \gamma(t)\gamma(s) \rangle \left[ V, \langle [\hat{U}_{\text{st}}(t, s)V\hat{U}_{\text{st}}^\dagger(t, s), \rho_{\text{st}}(t)] \rangle \right]$$

Simplified via Novikov's theorem for white noise

$$\frac{d}{dt}\rho(t) = -i[H_0(t), \rho(t)] - \frac{W^2}{2} [V, [V, \rho(t)]]$$

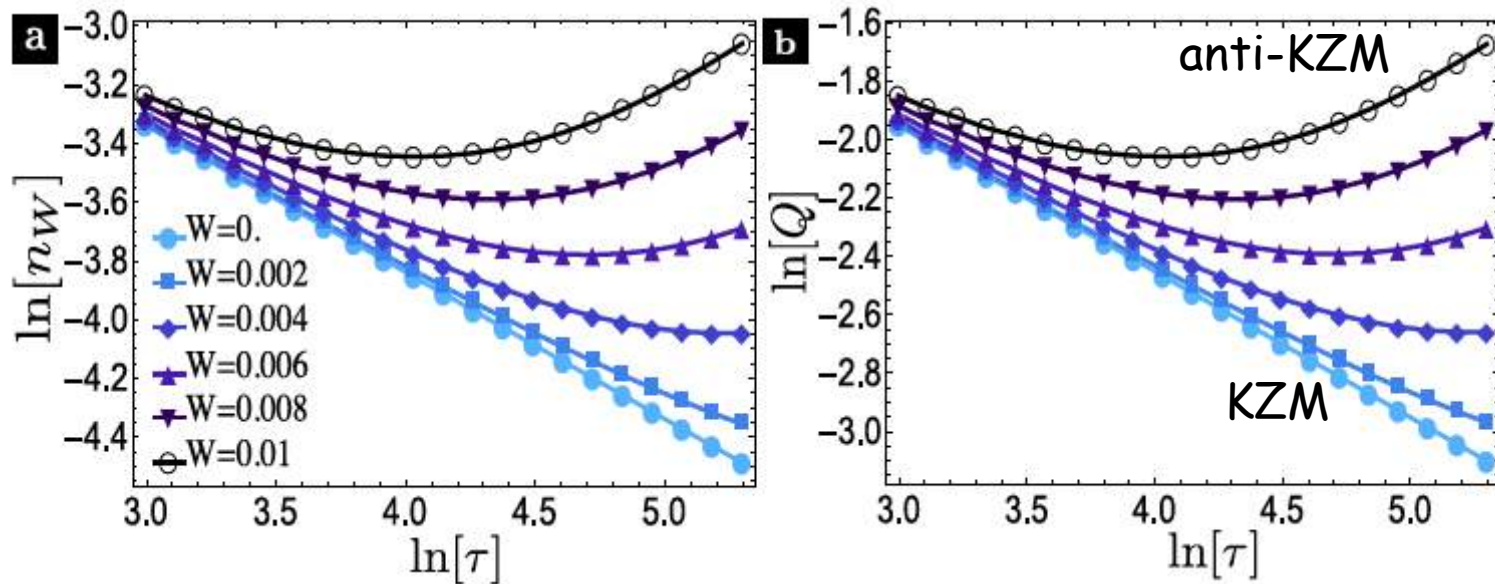
# Annealing dynamics

Density of excitations

$$n_W = 1 - \frac{1}{N} \sum_{k>0} \langle G_k(\tau) | \rho_k(t) | G_k(\tau) \rangle$$

Residual energy

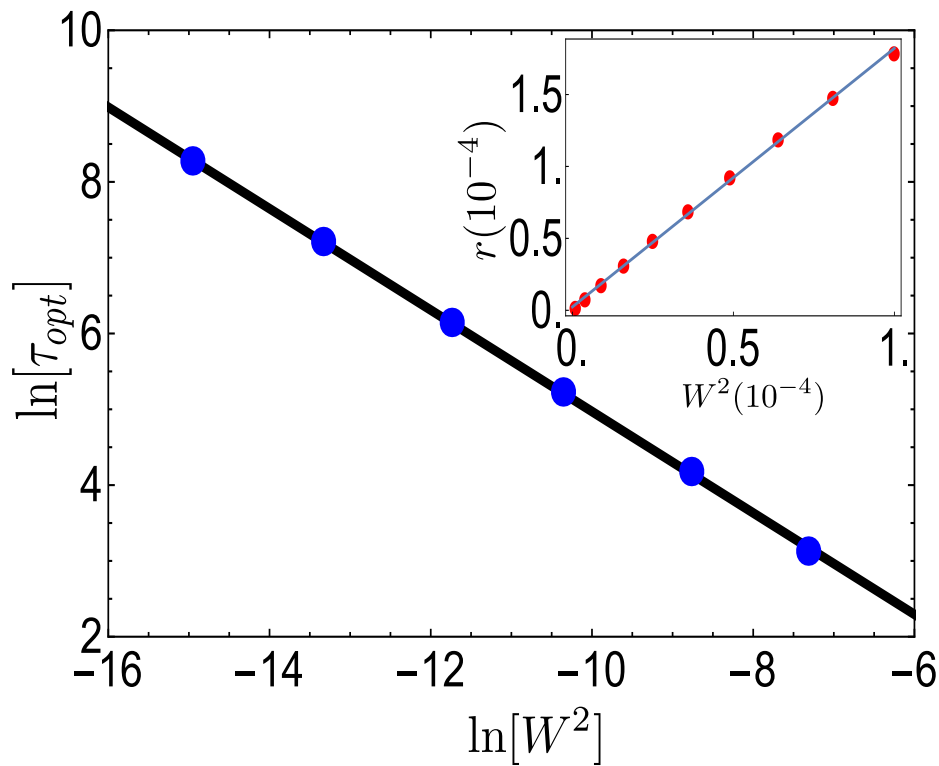
$$Q = [E(\tau) - E_{GS}(\tau)]/N$$



Noise-induced breakdown of adiabatic protocols  
Anti-Kibble-Zurek behavior



# Universality of optimal annealing time



Density of excitations

$$n_W \approx r\tau + c\tau^{-\beta}$$

Kibble-Zurek exponent

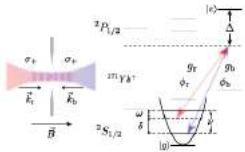
$$\beta = \frac{d\nu}{1 + z\nu}$$

Optimal annealing time

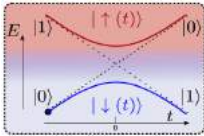
$$\tau_{opt} \propto (W^2)^{-1/(\beta+1)}$$

# Summary

- ◆ Kibble-Zurek scaling: long annealing times



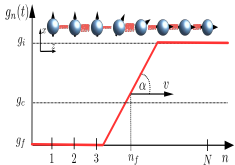
- ◆ Shortcuts to Adiabaticity provide a way out



- ◆ First exp test of KZM in the Quantum Regime



- ◆ Noise: Anti-Kibble Zurek behavior & optimal annealing time



- ◆ Inhomogeneous Quantum Annealing for AQC

## The Group

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**Armin Rahmani (British Columbia)**

**Marek Rams (Jagiellonian)**

**Masoud Mohseni (Google)**

**Enrique Solano (Bilbao)**

**Wojciech Zurek (LANL)**

**Chuang-Fen Li (Hefei)**

**Guan-Can Guo (Hefei)**



# Thanks for your attention!!

