Shortcuts to Adiabaticity and Quantum Speed Limits

Adolfo del Campo

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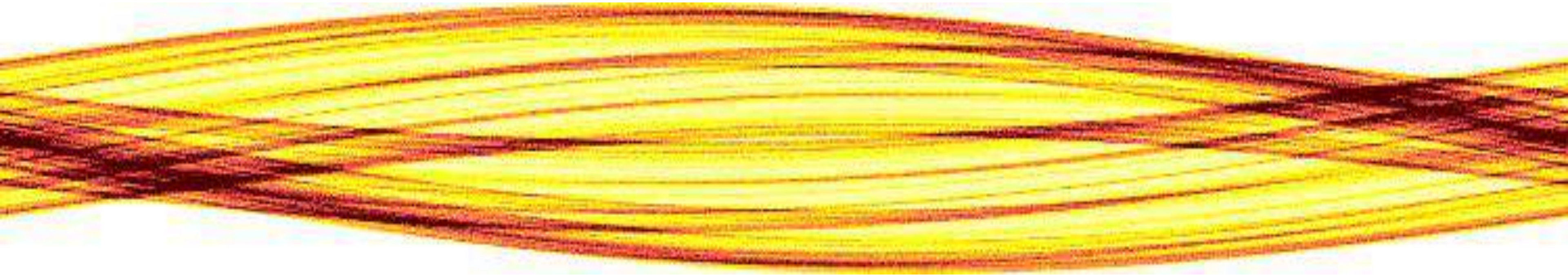
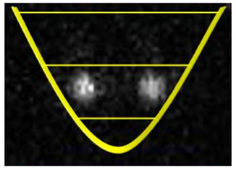
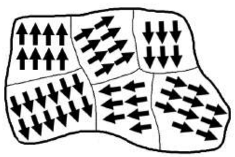


XVIII Giambiagi Winter School: Quantum Chaos & Control

July 25-29 2016, Buenos Aires



Talk 1: STA in noncritical systems



Talk 2: STA in critical systems

Talk 3: Quantum Speed Limits

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Talk 1: Contents

Techniques

- Inverting scaling laws
- Counterdiabatic driving
- Fast-Forward technique

Applications

- Fast transport
- Quantum thermodynamics

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Adiabatic dynamics

Consider driving a system Hamiltonian

$$\hat{H}_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$$

Born, Fock (1928); Kato, J. Phys. Soc. Jap. 5, 435 (1950), Avron & Elgart (1999)

Adiabatic dynamics

Consider driving a system Hamiltonian

$$\hat{H}_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$$

Write the adiabatic approximation

$$|\psi_n(t)\rangle = \exp\left[-\frac{i}{\hbar}\int_0^t E_n(s)ds - \int_0^t \langle n(s)|\partial_s n(s)\rangle ds\right]|n(t)\rangle$$

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Born, Fock (1928); Kato, J. Phys. Soc. Jap. 5, 435 (1950), Avron & Elgart (1999)

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Approximate solution of the TDSE

$$i\hbar\partial_t|\psi_n(t)\rangle \approx \hat{H}(t)|\psi_n(t)\rangle$$

Under SLOW driving

$$\hbar\frac{\langle n|\partial_t k\rangle}{E_n - E_k} \ll 1, \forall n \neq k$$

Born, Fock (1928); Kato, J. Phys. Soc. Jap. 5, 435 (1950), Avron & Elgart (1999)

Adiabatic dynamics

Slow driving of a system

Provides good control

No excitations

So, why shortcuts?

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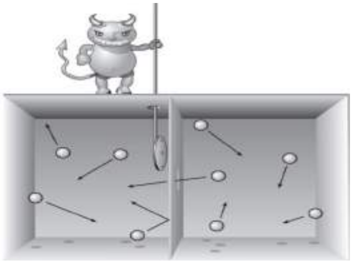
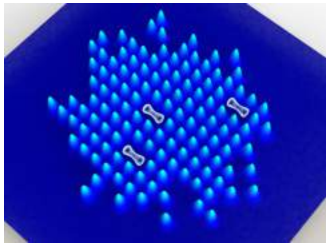
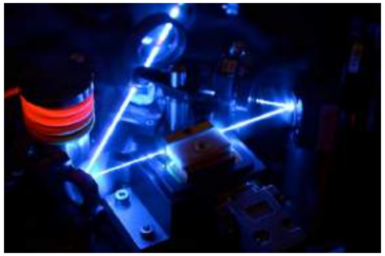
Well …



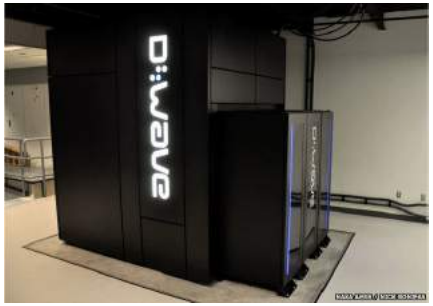
Quantum thermodynamics
energy conversion
ground state cooling



Quantum Simulation & Condensed Matter
defect suppression



Quantum Information
Quantum Optics
decoherence, noise



Adiabatic Quantum Computation

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Shortcuts to adiabaticity

Fast non-adiabatic process that mimics adiabatic dynamics
e.g. to prepare a state

[Review: Adv. At. Mol. Opt. Phys. **62**, 117 (2013)]

Processes: Expansion, transport, splitting, adiabatic passage, phase transitions, …

Systems: ultracold atoms, ions chains, quantum dots, spin systems, NVC, …

Experiments: Nice, NIST, Mainz, PTB, MPQ, Florence, Trento, Tsukuba, …

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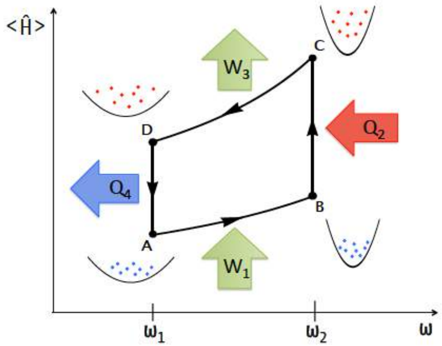
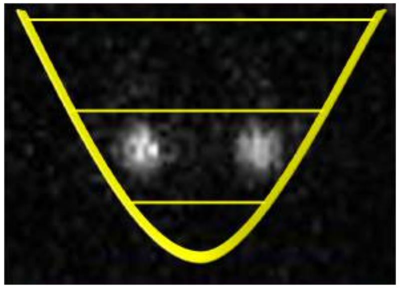
Shortcuts to adiabaticity



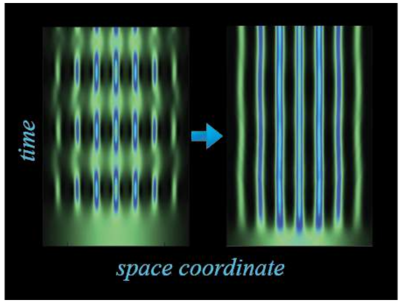
Quantum thermodynamics

Chen et al, PRL 104, 063002 (2010)
AdC & Boshier, Sci. Rep. 2, 648 (2012)
AdC, Goold, Paternostro Sci. Rep. 4, 6208 (2014)
Jaramillo, Beau, AdC, arXiv:1510.04633 (2016)
Beau, Jaramillo, AdC, Entropy 18, 168 (2016)



Transport

Deffner, Jarzynski, AdC PRX 4, 021013 (2014)
An, Lv, AdC, Kihwan Kim, arXiv:1601.05551

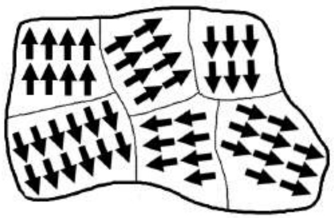


Loading optical lattice

Masuda, Nakamura, AdC PRL 113, 063003 (2014)

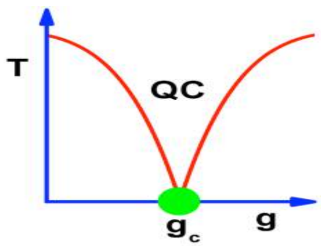
Quantum microscopy

AdC, EPL 96, 60005 (2011)
AdC, PRA 84, 031606(R) (2011)
AdC, PRL 111, 100502 (2013)



Topological Defect suppression

AdC et al. PRL105, 075701 (2010)
AdC et al. NJP 13, 083022 (2011)
Pyka et al. Nat. Commun. 4, 2291 (2013)
AdC, Kibble, Zurek, JPCM 25, 404210 (2013)
AdC & Zurek Int. J. Mod. Phys. A 29, 1430018 (2014)



Adiabatic crossing of quantum phase transition

AdC, Rams, Zurek PRL 109, 115703 (2012)
Saberi, Opatrný, Mølmer, AdC, PRA 90, 060301(R)
AdC & Sengupta, EPJ ST 224, 189 (2015)
Rams, Mohseni, AdC, TBS (2015)

And many other applications
(chemical rate processes, quantum logic gates, soliton dynamics, atom interferometry, …)

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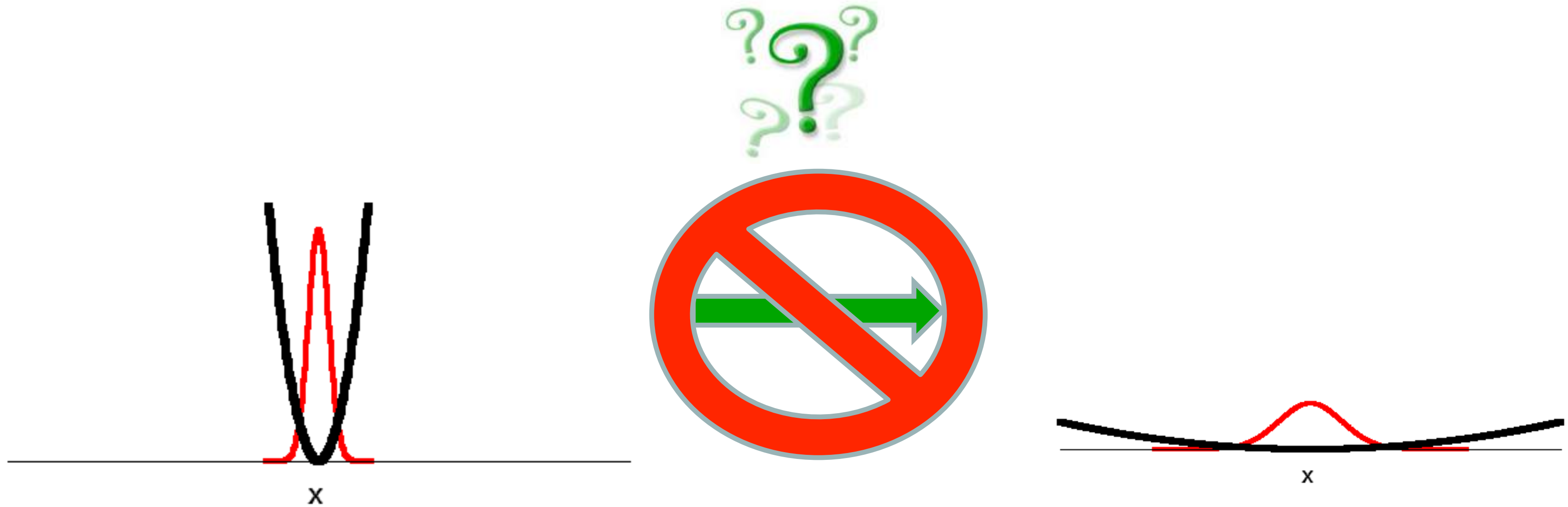
Inverting Scaling Laws

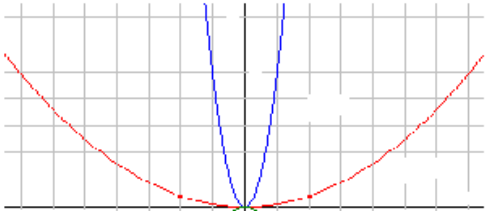
Inverting Scaling Laws



Standard expansion

Opening the trap

$$\omega(t) = \omega_i \left[1 + \frac{\omega_f - \omega_i}{\omega_i} \tanh \frac{t}{\tau} \right]$$



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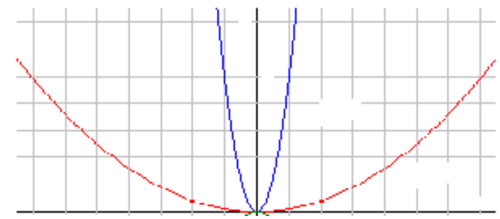
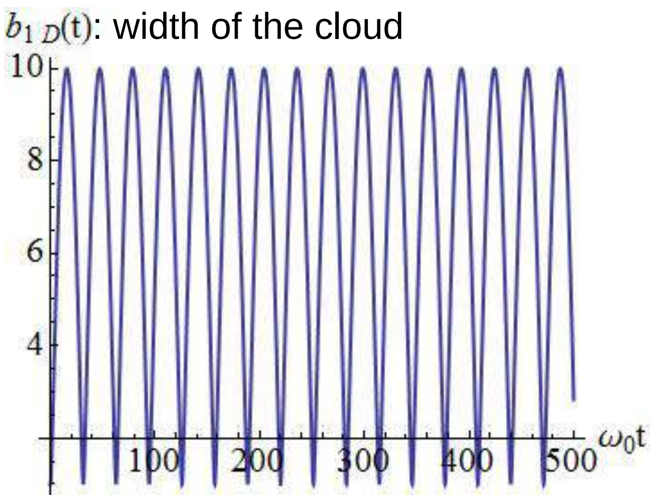
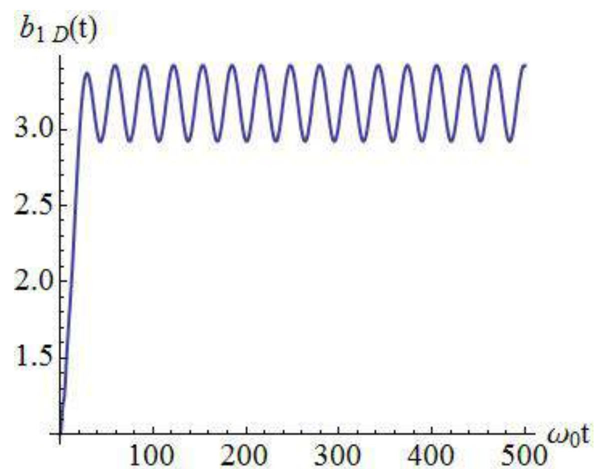
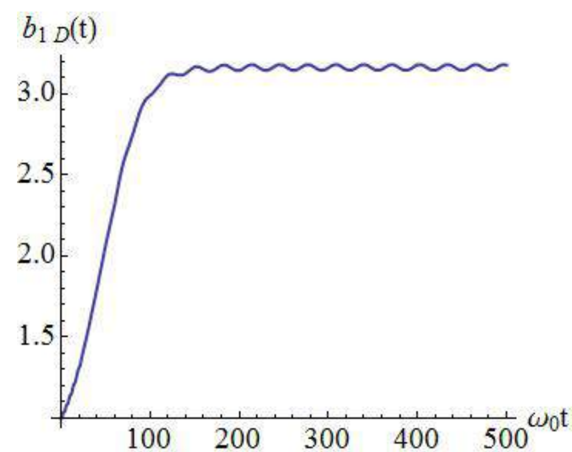


from sudden to adiabatic







Excitation of the breathing mode of the cloud

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Self-similar dynamics

1. Consider a time-dependent Hamiltonian harmonic oscillator

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega(t)^2 x^2$$

$$\hat{H}\phi_n(x) = E_n\phi_n(x)$$

2. Impose a self-similar dynamical ansatz

$$\phi(x,t) = \frac{1}{b(t)^{1/2}}\exp\left[i\frac{m\dot{b}(t)}{2\hbar b(t)}x^2 - i\int_0^t \frac{E_n}{b(s)^2}ds\right]\phi\left[\frac{x}{b(t)}, t=0\right]$$

3. Get the consistency equation: scaling factor as function of trap frequency

$$\ddot{b} + \omega^2(t)b = \omega_0^2/b^3$$

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Lewis & Riesenfeld J. Math. Phys. **10**,1458 (1969)
Chen et al., Phys. Rev. Lett. **104**, 063002 (2010)

Self-similar dynamics

1. Take a somewhat general many-body time-dependent Hamiltonian

$$\hat{\mathcal{H}}=\sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_i^{(D)}+\frac{1}{2}m\omega^2(t)\mathbf{x}_i^2\right]+\epsilon\sum_{i<j}\mathrm{V}(\mathbf{x}_{ij}) \qquad \mathbf{x}_i\in\mathbb{R}^D,\ \mathbf{x}_{ij}=\mathbf{x}_i-\mathbf{x}_j$$

With a potential satisfying $\mathrm{V}(\lambda\mathbf{x})=\lambda^{\alpha}\mathrm{V}(\mathbf{x})$

2. Impose a self-similar dynamical ansatz

$$\Phi\left(\{\mathbf{x}_i\},t\right)=\frac{1}{b^{D/2}}e^{i\sum_{i=1}^{N}\frac{m\mathbf{x}_i^2\dot{b}}{2b\hbar}-i\mu\tau(t)/\hbar}\Phi\left(\left\{\frac{\mathbf{x}_i}{b}\right\},0\right)$$

3. Get the consistency equations, i.e.

$$\ddot{b}+\omega^2(t)b=\omega_0^2/b^3 \qquad \epsilon(t)=b^{\alpha-2}$$

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del Campo, PRA **84**, 031606(R) (2011); PRL **111**, 100502 (2013)

Design of a shortcut to adiabaticity

1. Force the scaling ansatz to reduce to the initial and final states considered

Boundary conditions:

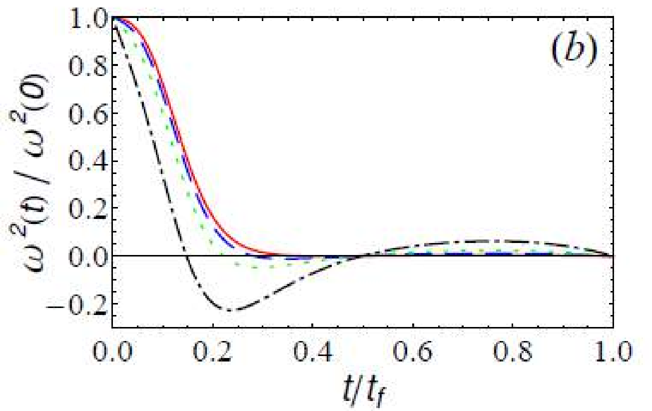
$$b(0) = 1, \quad \dot{b}(0) = 0, \quad \ddot{b}(0) = 0$$
$$b(\tau) = \sqrt{\frac{\omega_f}{\omega_0}}, \quad \dot{b}(\tau) = 0, \quad \ddot{b}(\tau) = 0$$

2. Determine an ansatz for the scaling factor (e.g. a polynomial)

$$b(t) = \sum_{j=0}^{5} a_j t^j$$

3. Find the driving frequency and coupling strength from the consistency equations

$$\ddot{b} + \omega^2(t) b = \omega_0^2 / b^3$$
$$\epsilon(t) = b^{\alpha-2}$$



Chen et al. Phys. Rev. Lett. **104**, 063002 (2010)
del Campo, PRA **84**, 031606(R) (2011)

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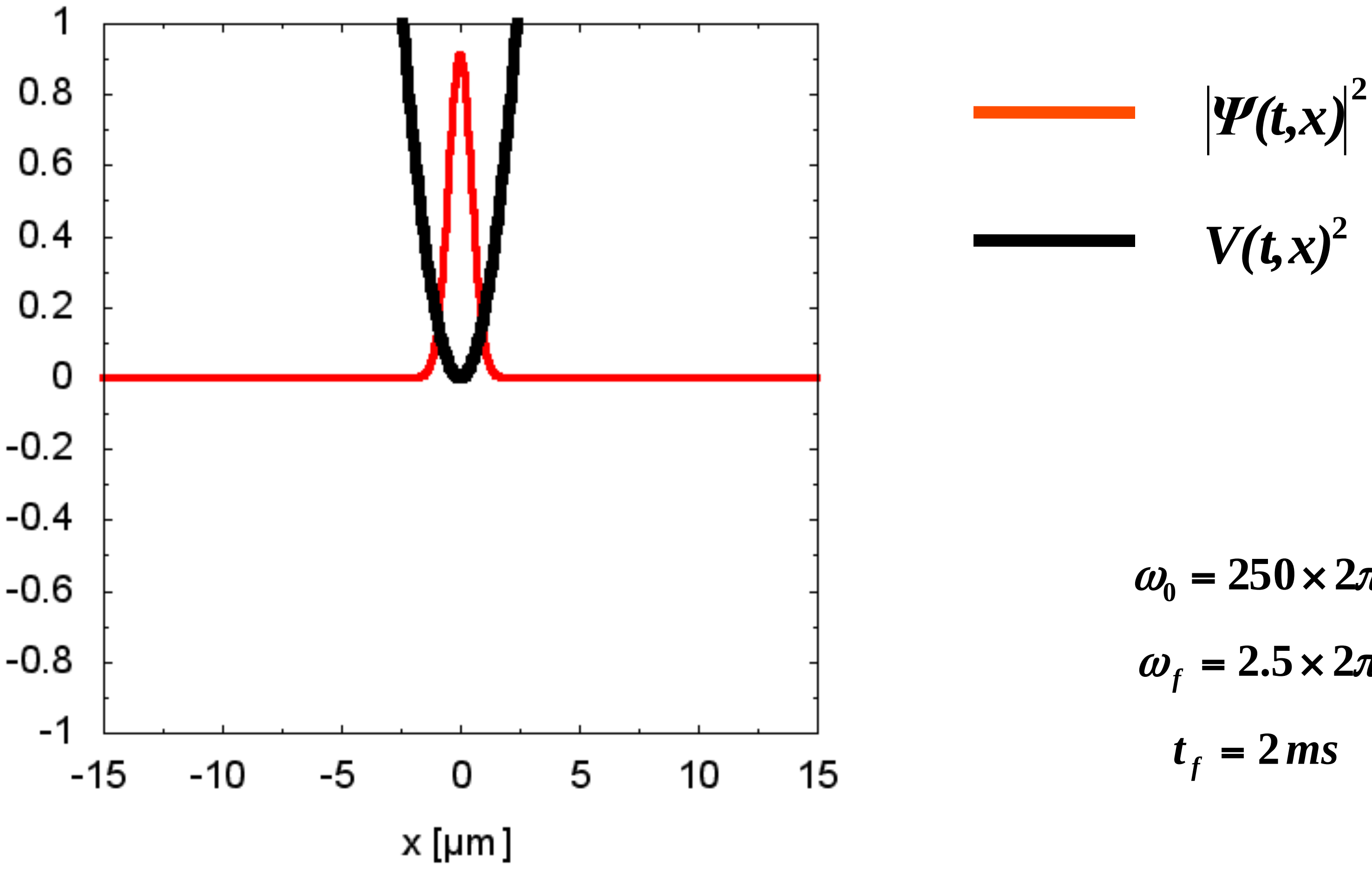
Example

Time Evolution:



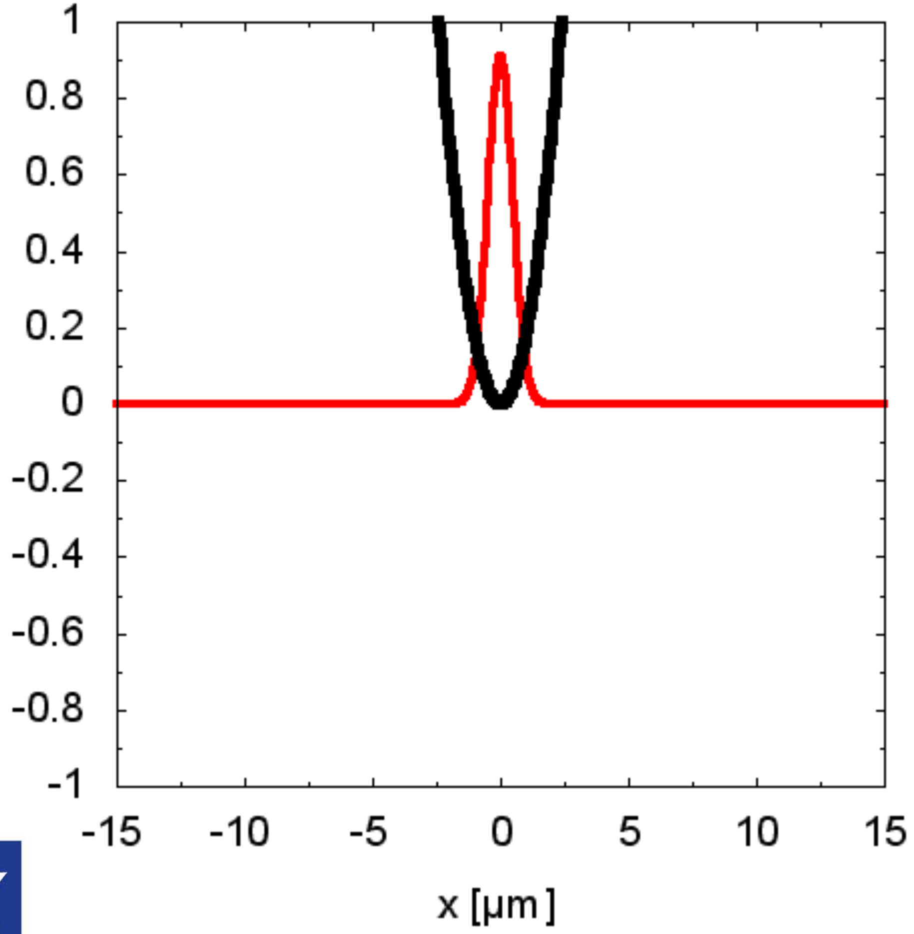
$$\omega_0 = 250 \times 2\pi\ Hz$$

$$\omega_f = 2.5 \times 2\pi\ Hz$$

$$t_f = 2\,ms$$

Example

Time Evolution:



$|\Psi(t,x)|^2$

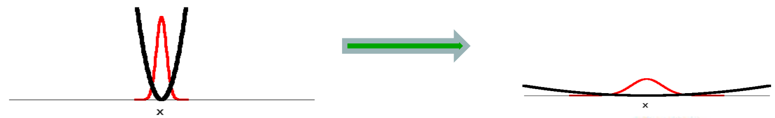
$V(t,x)^2$

$\omega_0 = 250 \times 2\pi \ Hz$

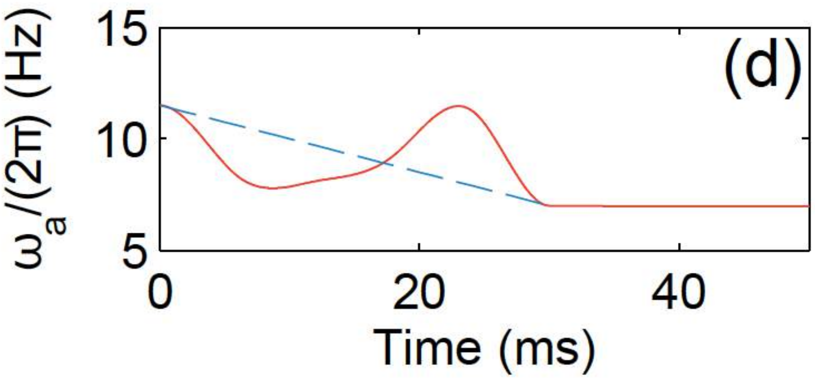
$\omega_f = 2.5 \times 2\pi \ Hz$

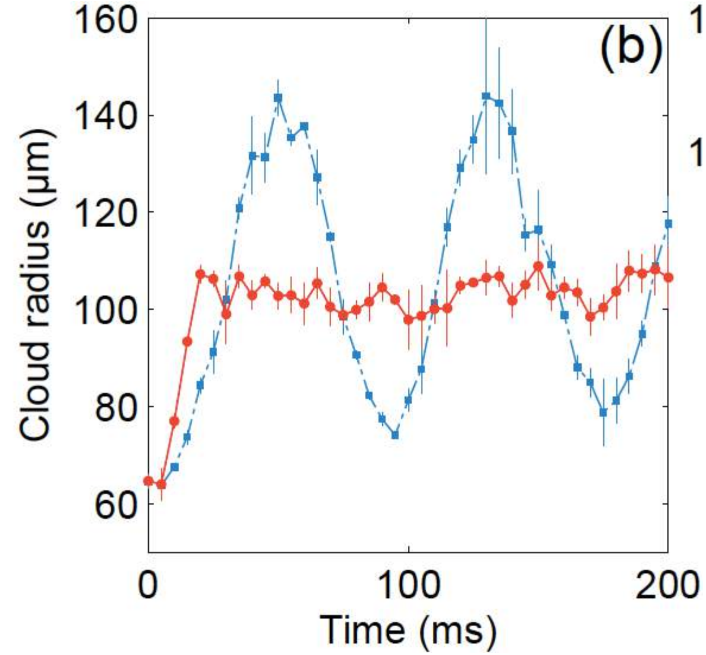
$t_f = 2 \, ms$

Experiments: Thermal cloud, BEC and 1D Bose gas



Shortcut vs standard expansion

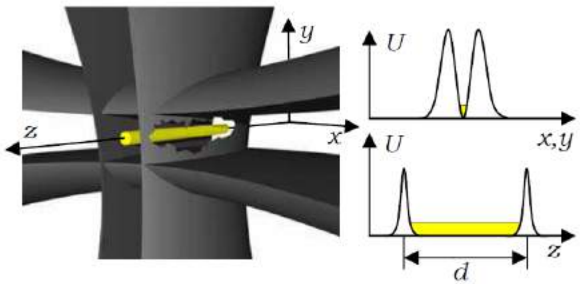




Experiments: 1D Bose gas
Rohringer et al. Sci. Rep. **5**, 9820 (2015)
Experiments: mean-field BEC
J.-F. Schaff et al. EPL **93**, 23001 (2011)
Experiments: single-particle
J.-F. Schaff et al. Phys. Rev. A **82**, 033430 (2010)

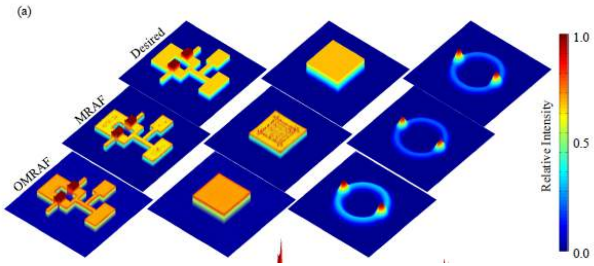
Theory (quantum fluids)
Chen et al. PRL **104**, 063002 (2010)
AdC PRA **84**, 031606(R) (2011)
AdC PRL **111**, 100502 (2013)

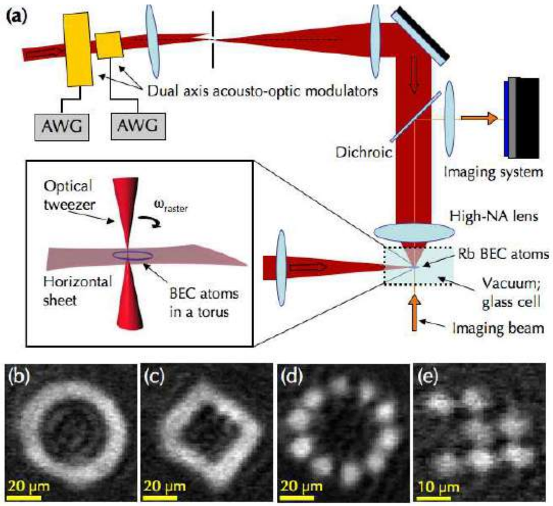
Nonharmonic traps? Boxes?



UT Austin all optical box
at Raizen's Lab
PRA, 71, 041604(R) (2005).

Cambridge's boxes
A. L. Gaunt, Z. Hadzibabic,
Sci. Rep. 2, 721 (2012)





Boshier's group at LANL
New J. Phys. 11, 043030 (2009)

+ implementations in atom chips

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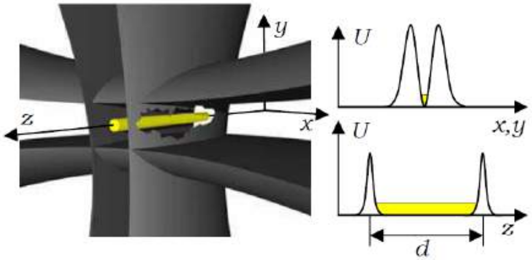
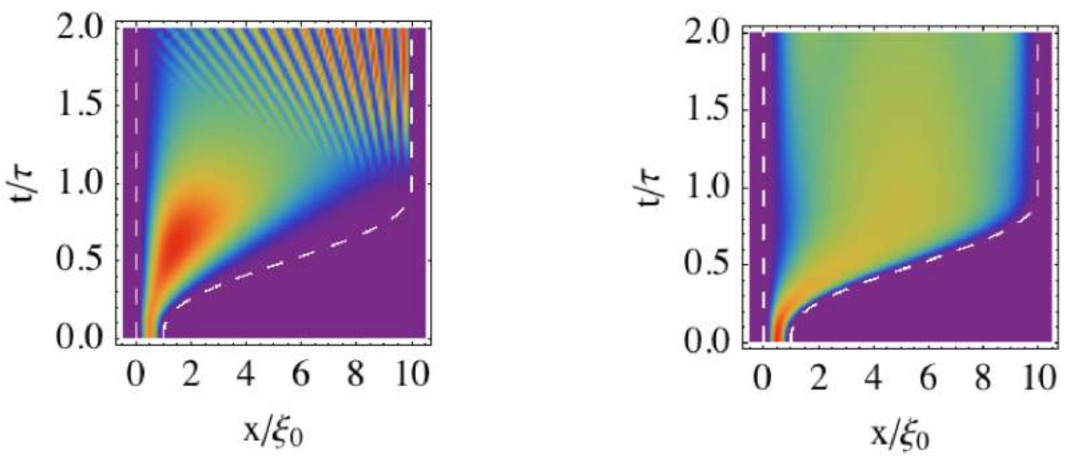
Quantum Piston



AdC & Boshier, Sci. Rep **2**, 648 (2012)
AdC, PRL **111**, 100502 (2013)

Quantum Piston



$$\Omega^2(t) = 0$$

$$\Omega^2(t) = -\frac{\ddot{\xi}(t)}{\xi(t)} \sim \frac{1}{\tau}$$

Counterdiabatic driving

Counterdiabatic driving

Consider driving a system Hamiltonian

$$\hat{H}_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$$

Write the adiabatic approximation

$$|\psi_n(t)\rangle = \exp\left[-\frac{i}{\hbar}\int_0^t E_n(s)ds - \int_0^t \langle n(s)|\partial_s n(s)\rangle ds\right]|n(t)\rangle$$

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Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

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Counterdiabatic driving

Consider driving a system Hamiltonian

$$\hat{H}_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$$

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$$|\psi_n(t)\rangle = \exp\left[-\frac{i}{\hbar}\int_0^t E_n(s)ds - \int_0^t \langle n(s)|\partial_s n(s)\rangle ds\right]|n(t)\rangle$$

Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

Yes, indeed!

$$\hat{H}(t) \equiv \hat{H}_0(t) + \hat{H}_1(t)$$

$$\hat{H}_1(t) = i\hbar\sum_n \left(|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle|n\rangle\langle n|\right)$$

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Counterdiabatic driving

\

Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

Yes, indeed!

$$\hat{H}(t) \equiv \hat{H}_0(t) + \hat{H}_1(t)$$

$$\hat{H}_1(t) = i\hbar \sum_{n\neq m}\sum_{m} \frac{|m\rangle\langle m|\partial_t \hat{H}_0|n\rangle\langle n|}{E_n(t) - E_m(t)}$$

Theory: Demirplak & Rice 2003; = M. V. Berry 2009 “Transitionless quantum driving”
CD inspired experiment for TLS: Morsch’s group Nature Phys. 2012; NVC: Suter’s group PRL 2013

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Counterdiabatic driving: applications

Counterdiabatic terms are often **nonlocal**

Search for experimentally-realizable local Unitarily equivalent Hamiltonians

$$\hat{H}' = U\hat{H}U^\dagger - i\hbar U\partial_t U^\dagger$$

RAP in Two level system (spin flip) $\hat{H}_1 \propto \sigma_y$ $\hat{H}_1' \propto \sigma_z$

Demirplak & Rice 2003 Bason et al 2012

Time-dependent harmonic oscillator $\hat{H}_1 \propto (xp + px)$ $\hat{H}_1' \propto x^2$

Muga el at 2010, Jarzynski 2013 Ibáñez et al 12, AdC 13

Transport of matter waves $\hat{H}_1 \propto p$ $\hat{H}_1' \propto x$

Deffner-Jarzynski-AdC 14

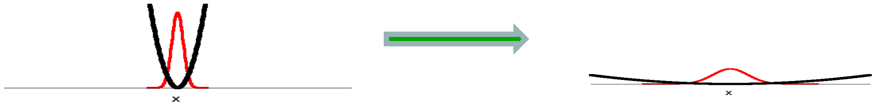
Theory: Demirplak & Rice 2003; M. V. Berry 2009
Experiment for TLS: Morsch's group Nature Phys. 2012; NVC: Suter's group PRL 2013

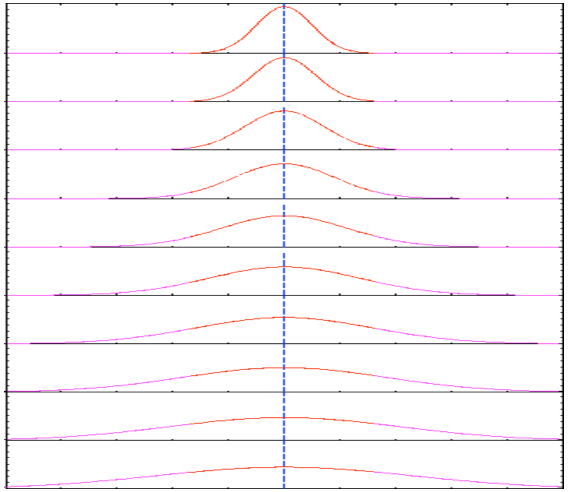
Quantum gases

Interacting quantum fluids

$$\hat{H}=\sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\nabla^2+\frac{1}{2}m\omega(t)^2\mathbf{r}_i^2\right]+\sum_{i<j}V(\mathbf{r}_i-\mathbf{r}_j)$$

Scaling-invariant dynamics when $V(\gamma\mathbf{r})=\gamma^{-2}V(\mathbf{r})$





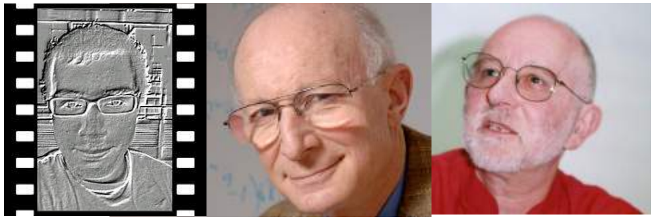
A. del Campo, PRL 111, 100502 (2013)

Quantum gases

Interacting quantum fluids

$$\hat{H} = \sum_{i=1}^{N} \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega(t)^2\mathbf{r}_i^2 \right] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Counterdiabatic Driving?

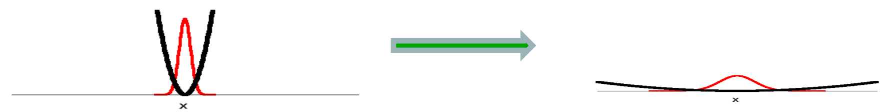


Spectral properties unavailable, even by numerical methods

A. del Campo, PRL **111**, 100502 (2013)

Quantum gases

More general case



$$\hat{H}_0(t) = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_{\mathbf{q}_i} + \frac{1}{2}m\omega^2(t)\mathbf{q}_i^2 + U(\mathbf{q}_i,t)\right] + \epsilon(t)\sum_{i<j}V(\mathbf{q}_i - \mathbf{q}_j)$$

$$\gamma(t) = \left[\frac{\omega(0)}{\omega(t)}\right]^{1/2} \qquad U(\mathbf{q},t) = \frac{1}{\gamma^2(t)}U\left(\frac{\mathbf{q}}{\gamma(t)},0\right), \qquad \mathrm{V}(\lambda\mathbf{q}) = \lambda^{-\alpha}\mathrm{V}(\mathbf{q})$$

A. del Campo, PRL 111, 100502 (2013)

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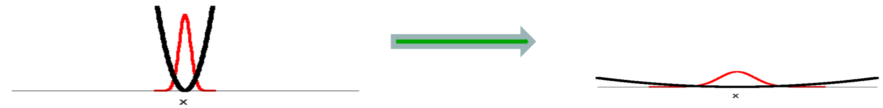
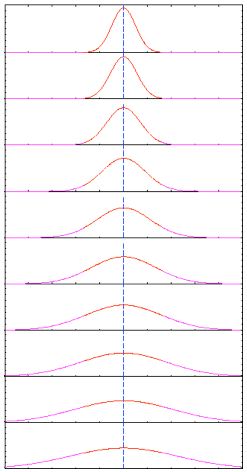
Quantum gases

More general case



$$\hat{H}_0(t) = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_{\mathbf{q}_i} + \frac{1}{2}m\omega^2(t)\mathbf{q}_i^2 + U(\mathbf{q}_i,t)\right] + \epsilon(t)\sum_{i<j}V(\mathbf{q}_i - \mathbf{q}_j)$$

$$\gamma(t) = \left[\frac{\omega(0)}{\omega(t)}\right]^{1/2} \qquad U(\mathbf{q},t) = \frac{1}{\gamma^2(t)}U\left(\frac{\mathbf{q}}{\gamma(t)},0\right), \qquad \mathrm{V}(\lambda\mathbf{q}) = \lambda^{-\alpha}\mathrm{V}(\mathbf{q})$$



Scaling ansatz

$$\Phi(t) = \gamma^{-\frac{ND}{2}}e^{-i\mu\tau(t)/\hbar}\Phi\left[\frac{\mathbf{q}_1}{\gamma(t)},\dots,\frac{\mathbf{q}_N}{\gamma(t)};0\right]$$

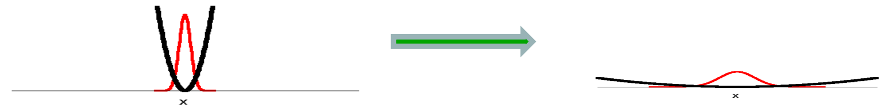
Nonlocal auxiliary Hamiltonian

$$\hat{H}_1 = -i\frac{\hbar\dot{\gamma}}{2\gamma}\sum_{i=1}^{N}(\mathbf{q}_i\partial_{\mathbf{q}_i} + \partial_{\mathbf{q}_i}\mathbf{q}_i)$$

A. del Campo, PRL 111, 100502 (2013)

Quantum gases

More general case



$$\hat{H}_0(t)=\sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_{\mathbf{q}_i}+\frac{1}{2}m\omega^2(t)\mathbf{q}_i^2+U(\mathbf{q}_i,t)\right]+\epsilon(t)\sum_{i<j}V(\mathbf{q}_i-\mathbf{q}_j)$$

$$\gamma(t)=\left[\frac{\omega(0)}{\omega(t)}\right]^{1/2}\qquad U(\mathbf{q},t)=\frac{1}{\gamma^2(t)}U\left(\frac{\mathbf{q}}{\gamma(t)},0\right),\qquad \mathrm{V}(\lambda\mathbf{q})=\lambda^{-\alpha}\mathrm{V}(\mathbf{q})$$

Scaling ansatz $\Phi(t)=\gamma^{-\frac{ND}{2}}e^{-i\mu\tau(t)/\hbar}\Phi\left[\frac{\mathbf{q}_1}{\gamma(t)},\dots,\frac{\mathbf{q}_N}{\gamma(t)};0\right]$

Unitary transformation $\mathcal{U}=\prod_{i=1}^{N}\exp\left(\frac{im\dot{\gamma}}{2\hbar\gamma}\mathbf{q}_i^2\right),\Phi(t)\to\Psi(t)=\mathcal{U}\Phi(t)$

LOCAL auxiliary Hamiltonian

$$\hat{\mathcal{H}}_1=-\frac{1}{2}m\frac{\ddot{\gamma}}{\gamma}\sum_{i=1}^{N}\mathbf{q}_i^2$$

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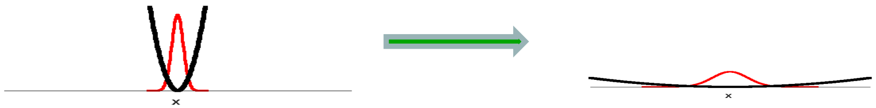
Adolfo del Campo: adolfo.delcampo@umb.edu

Quantum fluids: scaling laws & counterdiabatic driving

Family of interacting quantum fluids



$$\hat{H} = \sum_{i=1}^{N} \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega(t)^2\mathbf{r}_i^2 \right] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Scaling-invariant dynamics when $V(\gamma\mathbf{r}) = \gamma^{-2}V(\mathbf{r})$

Shortcut to adiabaticity = Fast motion video of adiabatic dynamics

Auxiliary Counterdiabatic Control => harmonic trap

$$\omega(t)^2 \rightarrow \Omega^2(t) = \omega^2(t) - \frac{3}{4}\frac{\dot{\omega}^2}{\omega^2} + \frac{1}{2}\frac{\ddot{\omega}}{\omega}\,.$$

A. del Campo, PRL **111**, 100502 (2013)

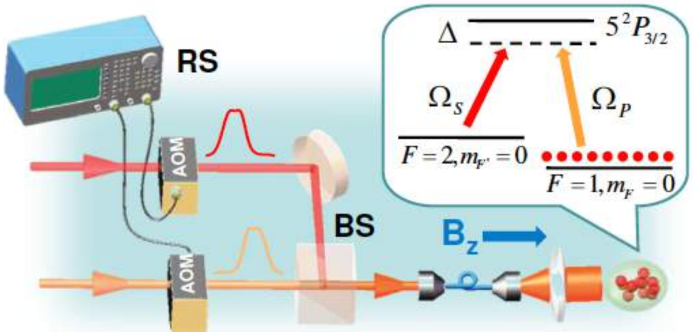
Adolfo del Campo: adolfo.delcampo@umb.edu

Counterdiabatic driving: Experiments

Experimental realization of stimulated Raman shortcut-to-adiabatic passage with cold atoms

Yan-Xiong Du[1], Zhen-Tao Liang[1], Yi-Chao Li[2], Xian-Xian Yue[1], Qing-Xian Lv[1], Wei Huang[1], Xi Chen[2, *], Hui Yan[1, *], Shi-Liang Zhu[3,1,4, *]
[1]*Guangdong Provincial Key Laboratory of Quantum Engineering and Quantum Materials, SPTE, South China Normal University, Guangzhou 510006, China*
[2]*Department of Physics, Shanghai University, Shanghai 200444 , China*
[3]*National Laboratory of Solid State Microstructures, School of Physics, Nanjing University, Nanjing 210093, China*
[4]*Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei 230026, China*

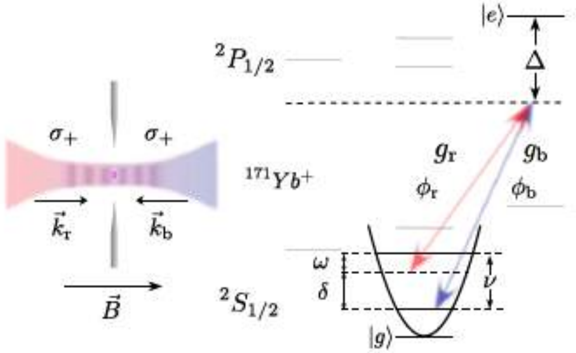
CD for 2 & 3 Level systems



Shortcuts to Adiabaticity by Counterdiabatic Driving in Trapped-ion Transport

Shuoming An,[1] Dingshun Lv,[1] Adolfo del Campo,[2] and Kihwan Kim[1]
[1]*Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, People's Republic of China*
[2]*Department of Physics, University of Massachusetts, Boston, MA 02125, USA*

CD for systems with Continuous Variables



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Fast-forward technique

Scale invariance is kind of classical

Really needed?

Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

Adolfo del Campo: adolfo.delcampo@umb.edu

Fast-forward technique

Consider the dynamics (mean-field)

$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + (\mathcal{V}+\mathcal{V}_{\rm au})\Psi + g|\Psi|^2\Psi,$$

Ansatz for the evolution

$$\Psi(\mathbf{q},t) = \psi[\mathbf{q},R(t)]e^{i\phi(\mathbf{q},t)}e^{-\frac{i}{\hbar}\int_0^t \mu[R(t')]dt'}$$

where

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \mathcal{V}\psi + g|\psi|^2\psi = \mu\psi.$$

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Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

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where

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \mathcal{V}\psi + g|\psi|^2\psi = \mu\psi.$$

Substituting ansatz, separating real and imaginary parts

$$\mathcal{V}_{\rm au}({\rm q},t) = -\frac{\hbar^2}{2m}(\nabla\phi)^2 - \hbar\partial_t\phi$$

$$\nabla^2\phi + 2\nabla\ln\psi\cdot\nabla\phi + \frac{2m}{\hbar}\dot{R}\partial_R\ln\psi = 0$$

determine the auxiliary driving potential

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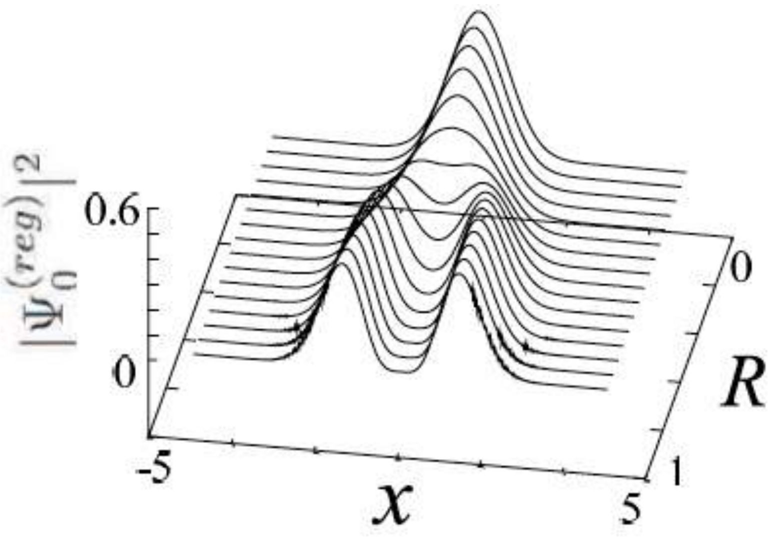
Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

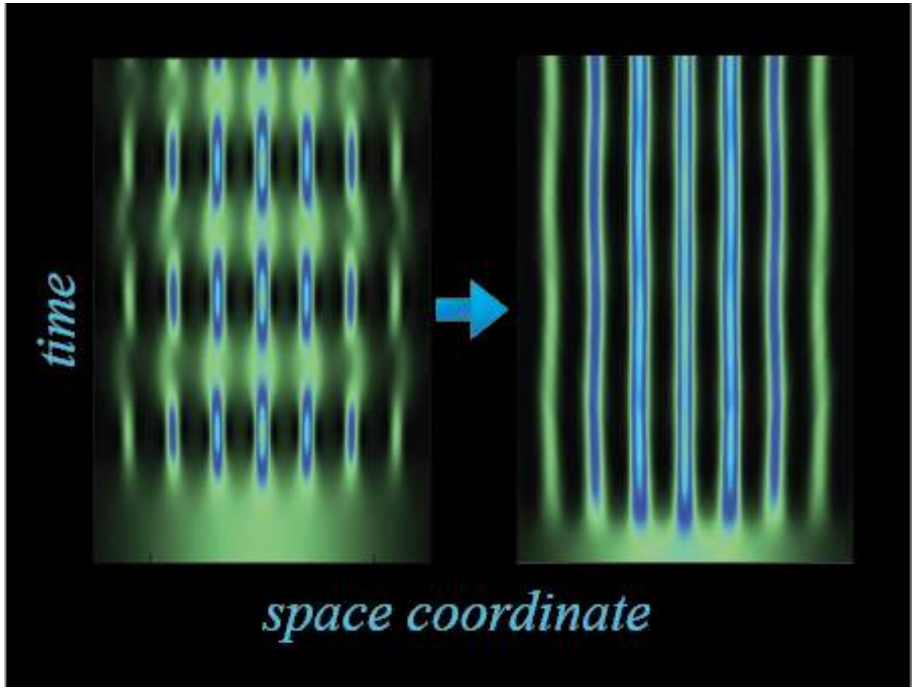
Examples

Matter wave splitting



Ground-state loading in an optical lattice

Auxliary potential ≈ bichromatic lattice



Masuda & Nakamura,
Proc. R. Soc. A **466**, 1135 (2010)
Torrontegui et al
PRA **87**, 033630 (2013)

Masuda, Nakamura, del Campo
PRL **113**, 063003 (2014)

$$\mathcal{V}_{\rm app}(q,t) = U_1(t)\sin^2(k_L q) + U_2(t)\sin^2(2k_L q)$$

Adolfo del Campo: adolfo.delcampo@umb.edu

Fast-forward technique

Scale invariance is kind of classical

Really needed?

Protocols become energy/state dependent

Critical systems:
AdC, Rams, Zurek PRL **109**, 115703 (2012)
Saberi, Opatrný, Mølmer, AdC PRA **90**, 060301(R) (2014)
Optical lattices:
Masuda, Nakamura, AdC PRL **113**, 063003 (2014)

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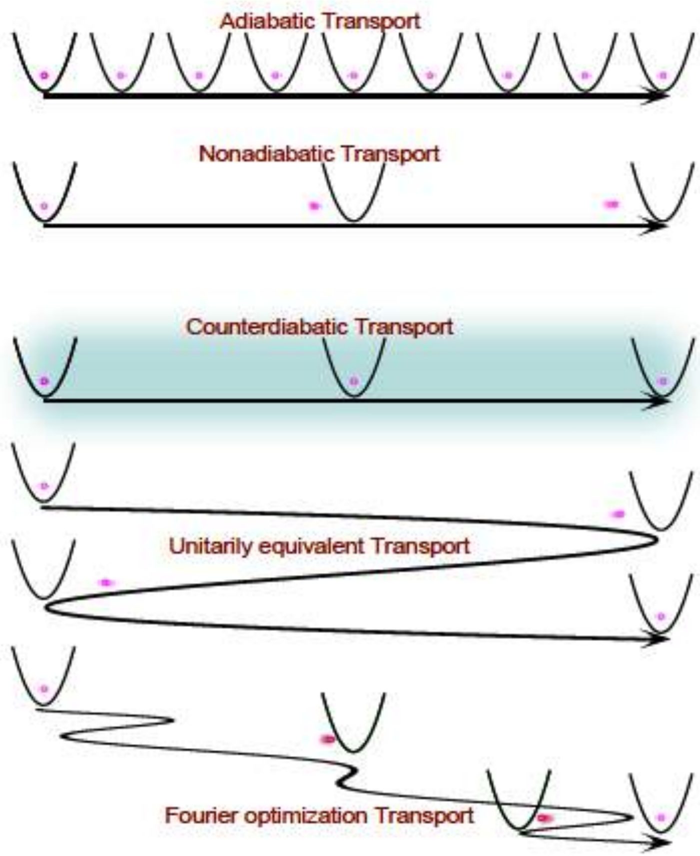
Part II: Applications
Shortcuts to adiabatic transport



Shuoming An, Dingshun Lv, AdC, Kihwan Kim, arXiv:1601.05551

Counterdiabatic transport

Transport of quantum fluids

$$\hat{\mathcal{H}} = \sum_{i=1}^{N}\Big[-\frac{\hbar^2}{2m}\Delta^2 + U[\mathbf{r}_i - \mathbf{f}(t)]\Big] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j),$$

Scale-invariant evolution of an initial stationary state

$$\Phi(\mathbf{r}_1, \ldots, \mathbf{r}_N; t) = e^{-i\mu t/\hbar}\Phi\left[\mathbf{r}_1 - \mathbf{f}(t), \ldots, \mathbf{r}_N - \mathbf{f}(t); 0\right]$$

NONLOCAL counterdiabatic term

$$\hat{\mathcal{H}}_1 = -i\hbar\sum_{i=1}^{N}\dot{\mathbf{f}}\partial_{\mathbf{r}_i} = \sum_{i=1}^{N}\dot{\mathbf{f}}\cdot\mathbf{p}_i.$$

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S. Deffner, C. Jarzynski, A. del Campo, PRX 4, 021013 (2014)

Counterdiabatic transport

Transport of quantum fluids

$$\hat{\mathcal{H}} = \sum_{i=1}^{N}\Big[-\frac{\hbar^2}{2m}\Delta^2 + U[\mathbf{r}_i - \mathbf{f}(t)]\Big] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j),$$

Scale-invariant evolution of an initial stationary state

$\Phi(\mathbf{r}_1, \ldots$

LOCAL CD

via unitary tr

Single-particle:
S. Masuda, K. Nakamura, Proc. R. Soc. A 466, 1135 (2010)
E. Torrontegui et al, Phys. Rev. A83,013415 (2011).
Many-particle:
S. Masuda, Phys. Rev. A 86, 063624 (2012)

$$\hat{\mathcal{H}}_{\rm CD} = \hat{\mathcal{H}} - \sum_{i=1}^{N} m\ddot{\mathbf{f}}\cdot\mathbf{r}_i$$

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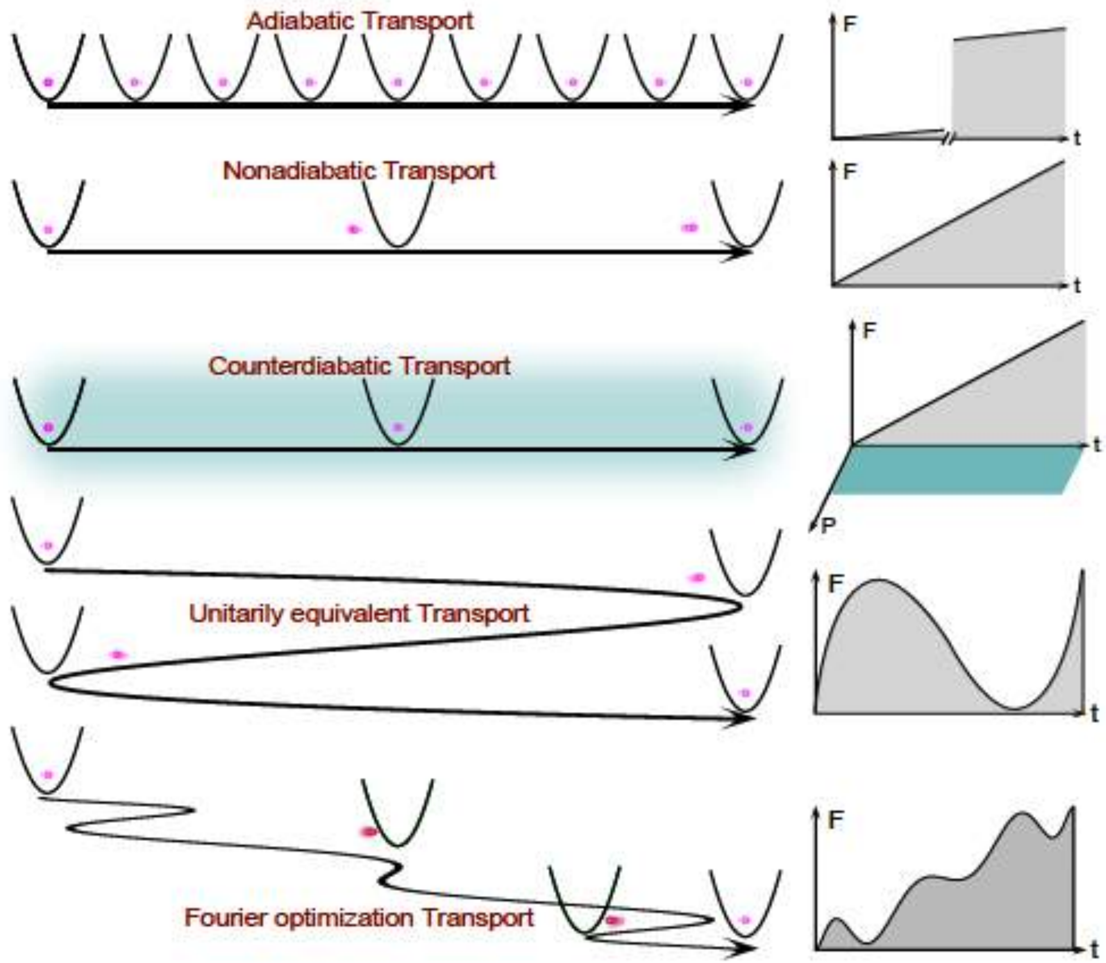
S. Deffner, C. Jarzynski, A. del Campo, PRX 4, 021013 (2014)

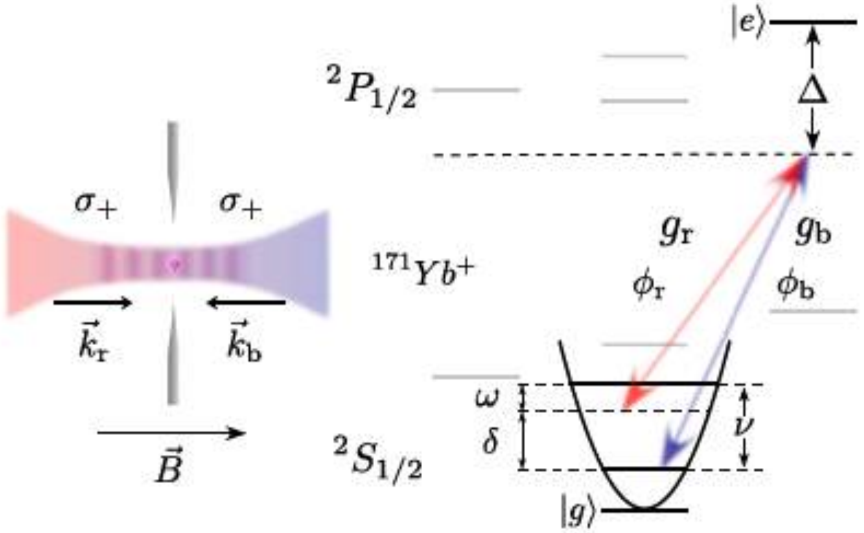
Shortcuts to Adiabaticity by Counterdiabatic Driving in Trapped-ion Transport

Shuoming An,[1] Dingshun Lv,[1] Adolfo del Campo,[2] and Kihwan Kim[1]
[1] *Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, People's Republic of China*
[2] *Department of Physics, University of Massachusetts, Boston, MA 02125, USA*



Recipe for a dragged harmonic oscillator

- Single trapped $^{171}Yb^{+}$ ion
- Raman beams exert the force
- Dipole approximation
- Rotating Wave approximation (RWA)
- Lamb-Dicke regime



$$\hat{H}_{\rm eff} = \hat{p}^2/2m + m\omega^2\hat{x}^2 + f(t)\hat{x} \qquad \hat{H}_{\rm CD} = -\frac{\dot{f}(t)}{m\omega^2}\hat{p}$$

$$\hat{H}_{\rm eff} = f(t)x_0\left(\hat{a}e^{-i(\omega t+\phi} + \hat{a}^\dagger e^{+i(\omega t+\phi}\right)$$

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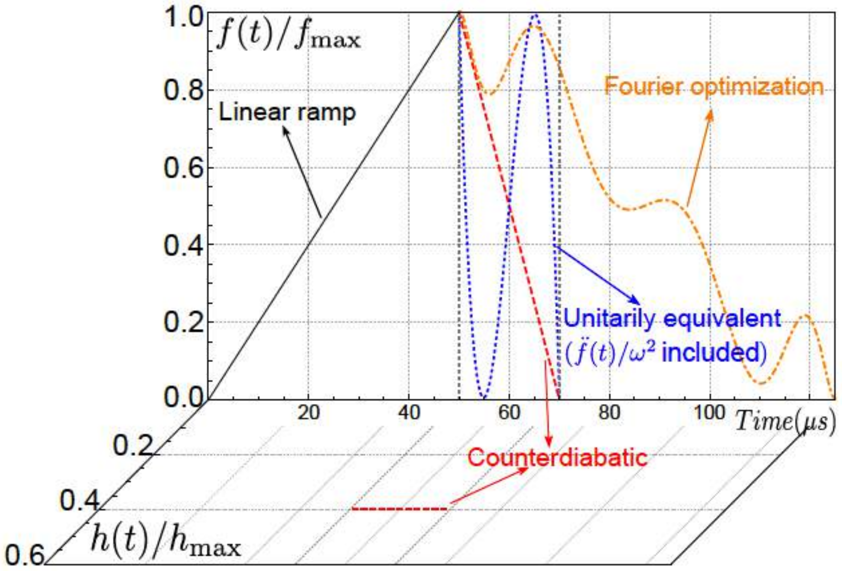
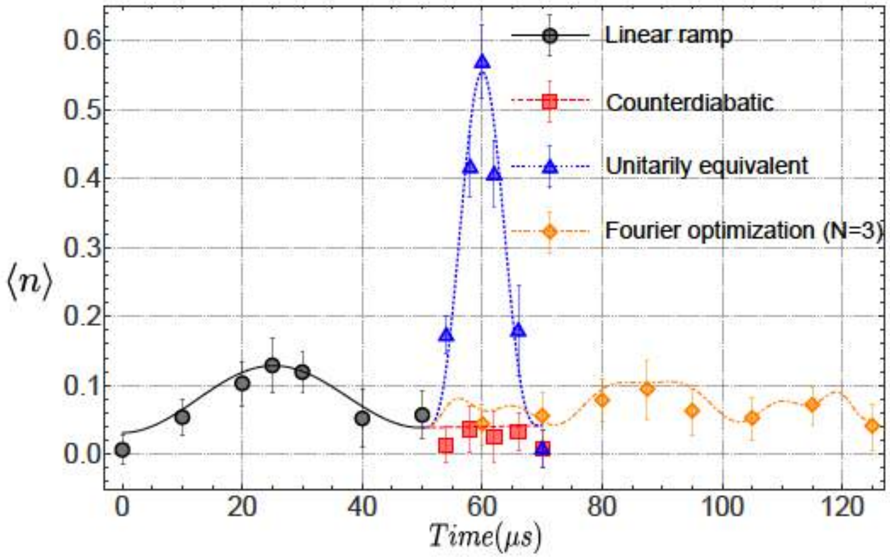
Comparison of transport protocols





Supremacy of counterdiabatic driving

Shortcuts to Adiabaticity by Counterdiabatic Driving in Trapped-ion Transport

Shuoming An,[1] Dingshun Lv,[1] Adolfo del Campo,[2] and Kihwan Kim[1]

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[2] *Department of Physics, University of Massachusetts, Boston, MA 02125, USA*

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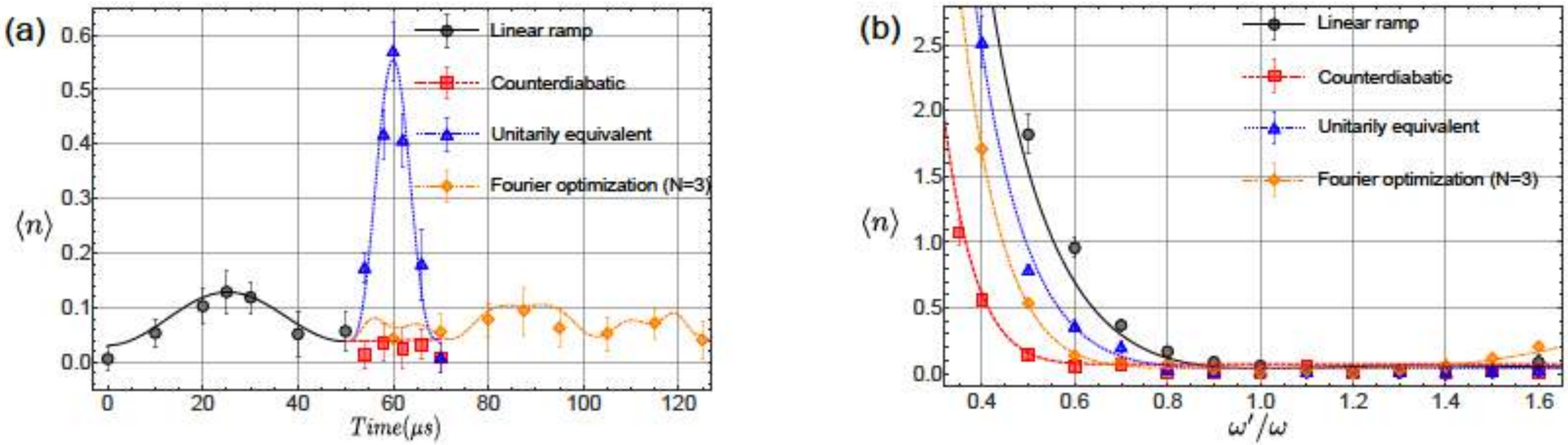
Robustness against trap frequency errors



Supremacy of counterdiabatic driving

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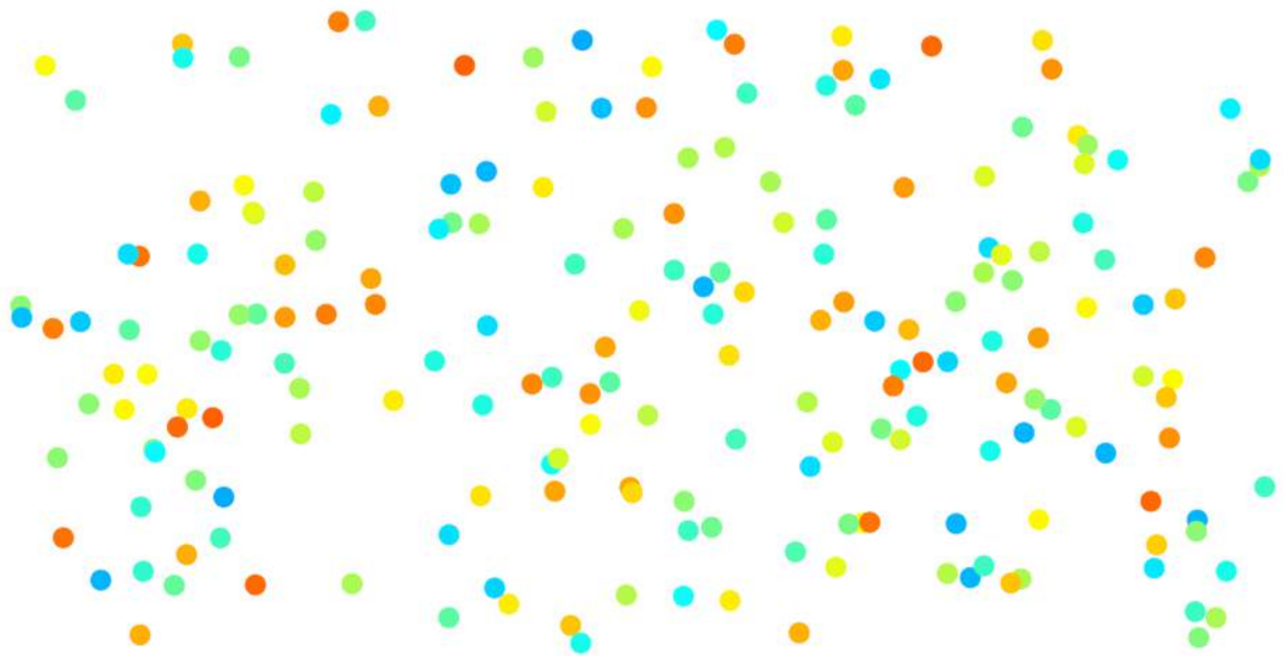
Part II: Applications

Shortcuts to adiabaticity in quantum thermodynamics

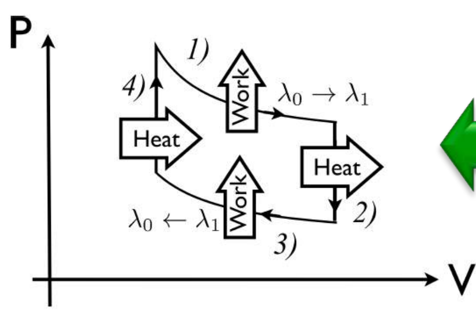


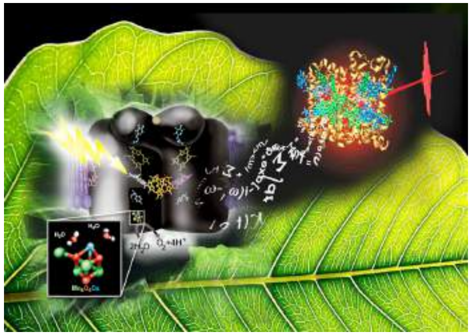
Adolfo del Campo: adolfo.delcampo@umb.edu

Quantum Heat Engines: Towards Green Quantum Energy

Optimal energy consumption and conversion

Equivalence Quantum engines & Photocells





Photosynthetic reaction center as a quantum heat engine

Konstantin E. Dorfman[a,b,c,1], Dmitri V. Voronine[a,b,1], Shaul Mukamel[c], and Marlan O. Scully[a,b,d]

[a]Texas A&M University, College Station, TX 77843-4242; [b]Princeton University, Princeton, NJ 08544; [c]University of California, Irvine, CA 92697-2025; and [d]Baylor University, Waco, TX 76798

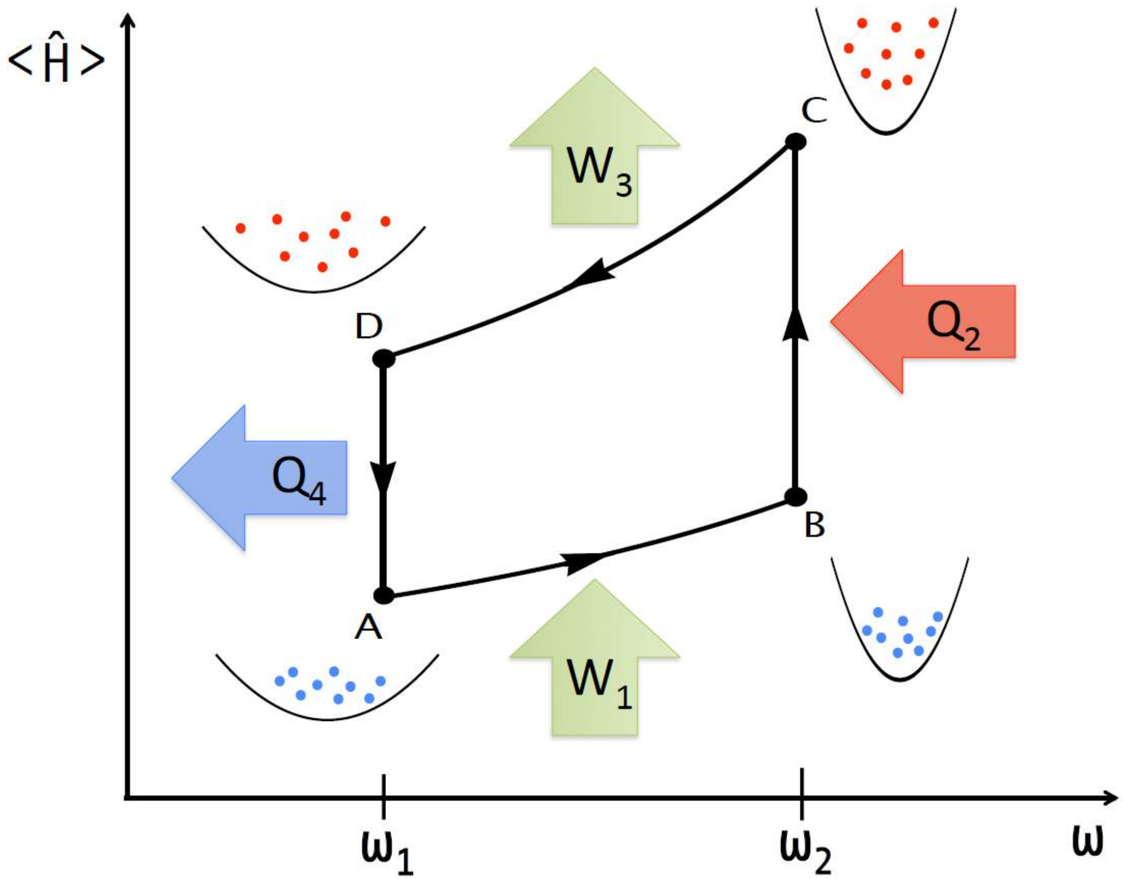
PNAS

PRL 111, 253601 (2013) PHYSICAL REVIEW LETTERS week ending 20 DECEMBER 2013

Efficient Biologically Inspired Photocell Enhanced by Delocalized Quantum States

C. Creatore,[1,*] M. A. Parker,[1] S. Emmott,[2] and A. W. Chin[1]
[1]*Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom*
[2]*Microsoft Research, Cambridge CB1 2FB, United Kingdom*

Quantum Heat Engines (e.g. Otto Cycle)



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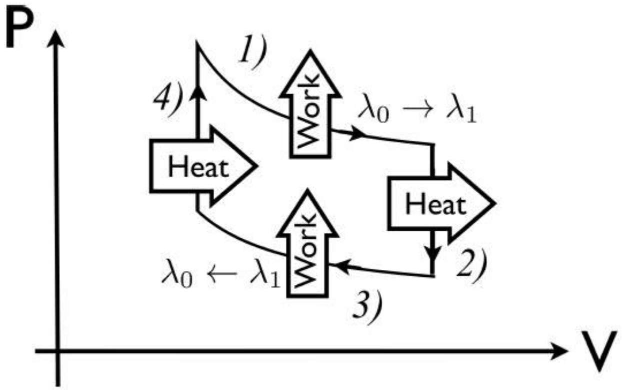
Efficiency vs Power



Quantum efficiency

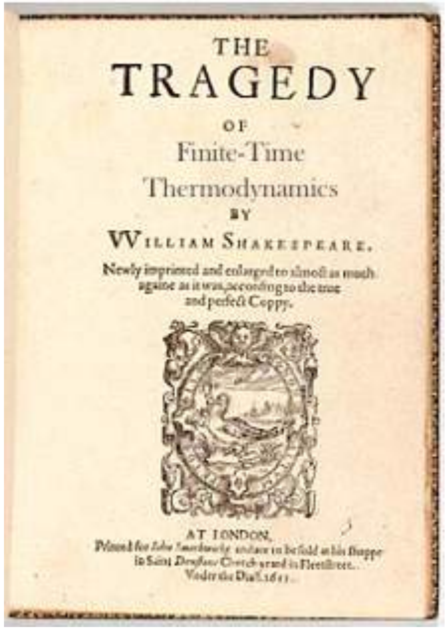
$$\eta = -\frac{\langle W_1 \rangle + \langle W_3 \rangle}{\langle Q_2 \rangle}$$

Nonadiabatic Efficiency e.g. of a single-particle Otto cycle

$$\eta = 1 - \frac{\omega_1}{\omega_2} f[\omega(t)] \leq 1 - \frac{\omega_1}{\omega_2}$$

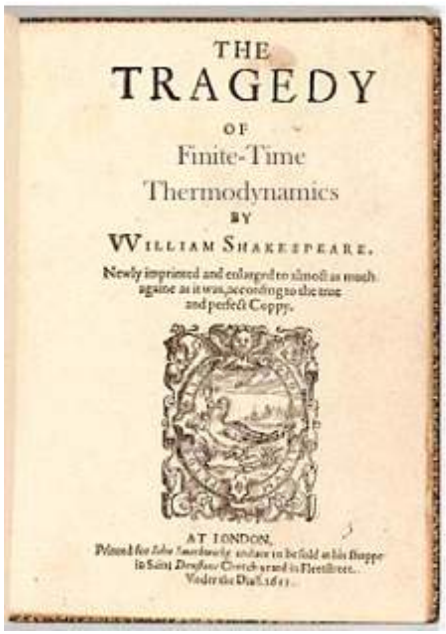
Essence of finite-time thermodynamics:

Trade-off between efficiency and power

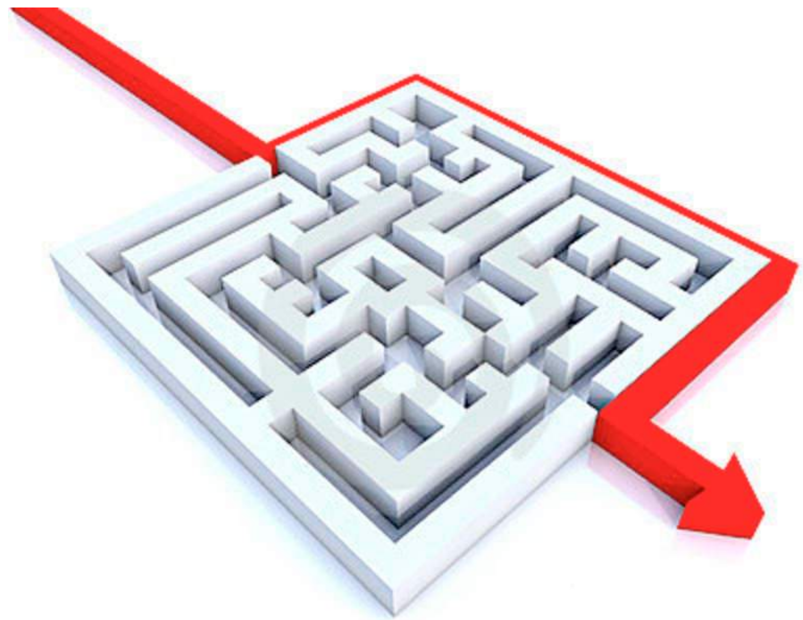


Adolfo del Campo: adolfo.delcampo@umb.edu

Shortcuts as a way out of the tragedy



Shortcuts to adiabaticity



- AdC, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014) (single-particle)
- M. Beau, J. Jaramillo, AdC, Entropy 18, 168 (2016) (many-particle)

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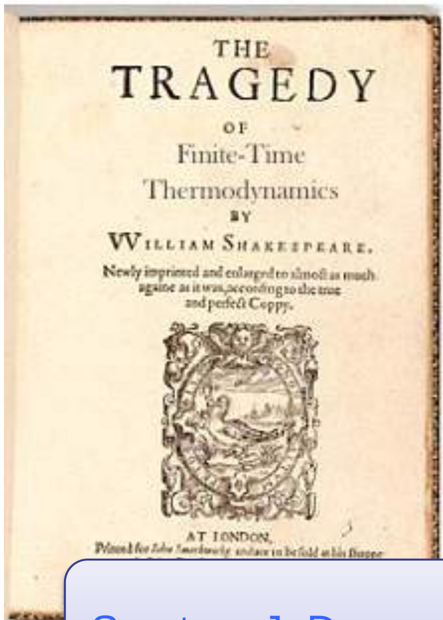
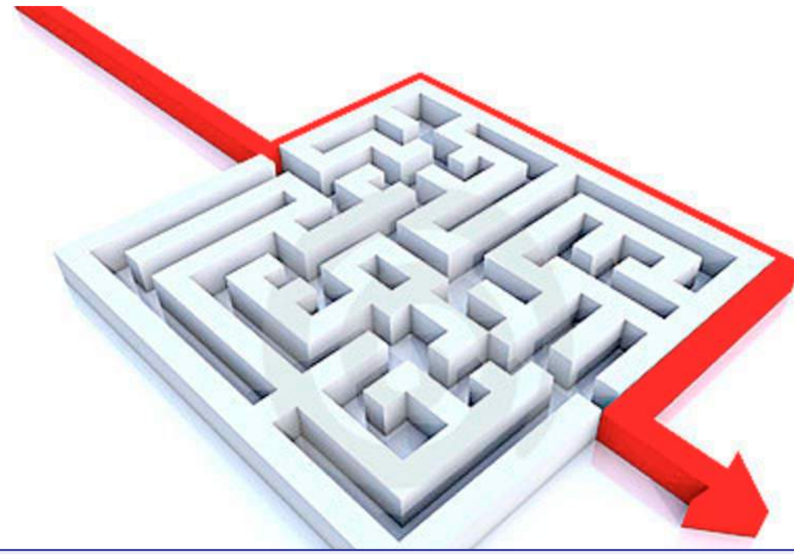
Shortcuts as a way out of the tragedy

Shortcuts to adiabaticity





See too: J. Deng et al., Phys. Rev. E **88**, 062122 (2013) (single-particle)

- AdC, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014) (single-particle)
- M. Beau, J. Jaramillo, AdC, Entropy **18**, 168 (2016) (many-particle)

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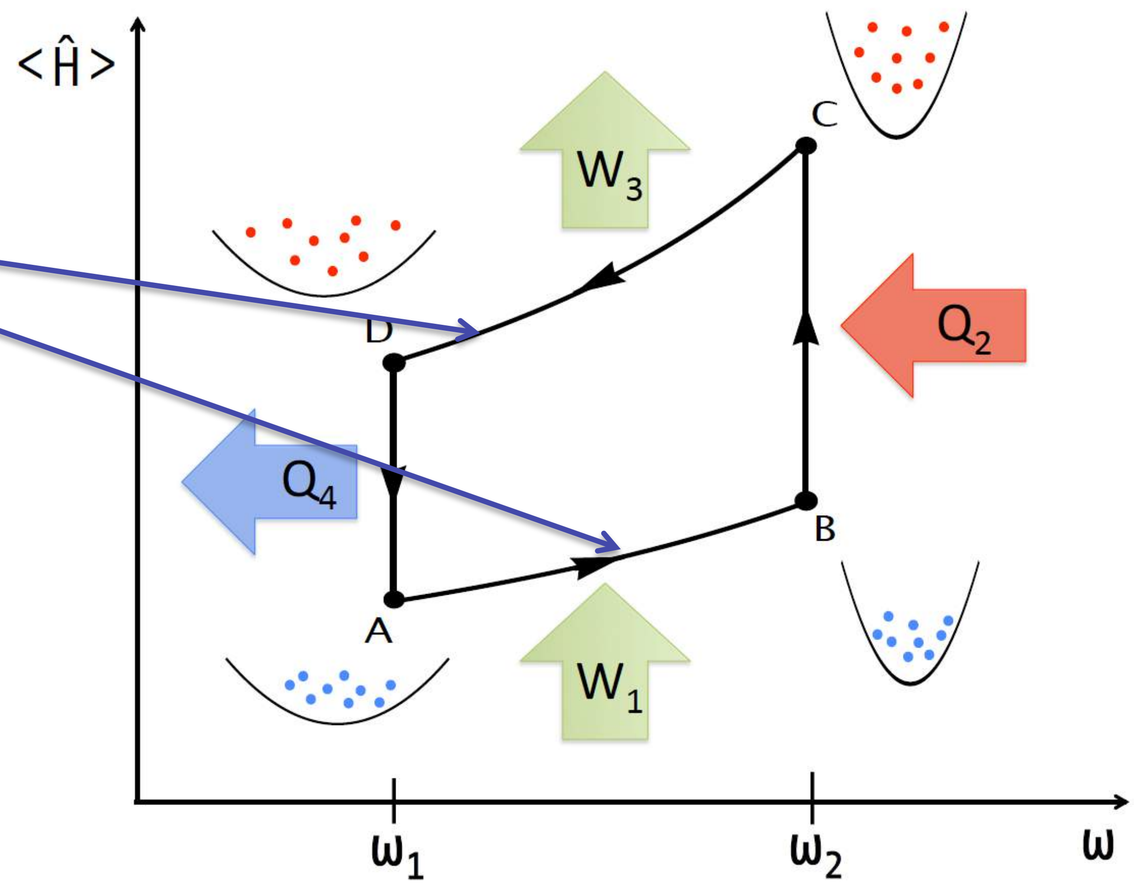
Quantum Heat Engines (e.g. Otto Cycle)

STA to expansion and compression



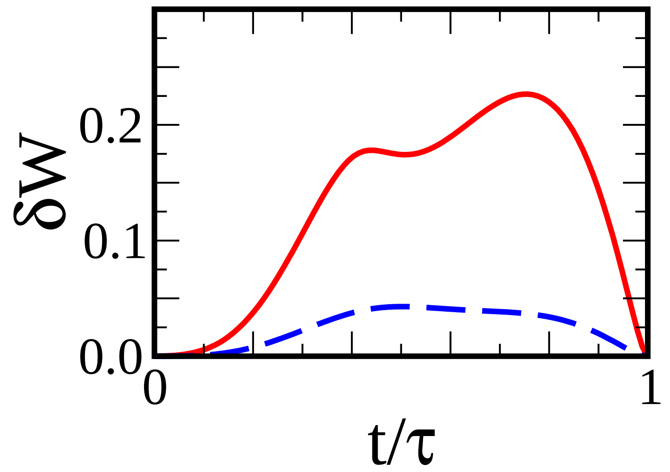
Superadiabatic quantum engine

Cycle with tunable power andmaximum efficiency (zero friction)

$$\mathcal{E}_{\max} = 1 - \frac{\omega(\tau)}{\omega(0)}$$

Initial state thermal

$$\delta W = \langle W \rangle - \langle W_{\rm ad} \rangle = \frac{1}{\beta_t} S(\rho_t || \rho_t^{\rm ad}), \qquad \rho_t^{\rm ad} = \sum_n p_n^0 |n(t)\rangle\langle n(t)|$$



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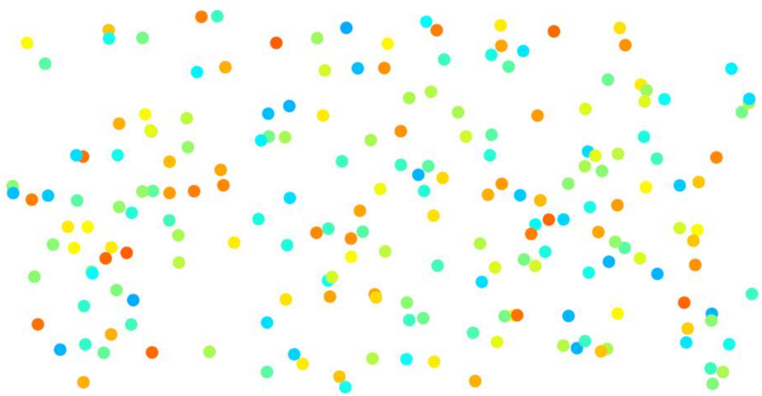
Many-particle QHE

Single N-particle engine vs N single-particle engines?

What substance is optimal as working medium?



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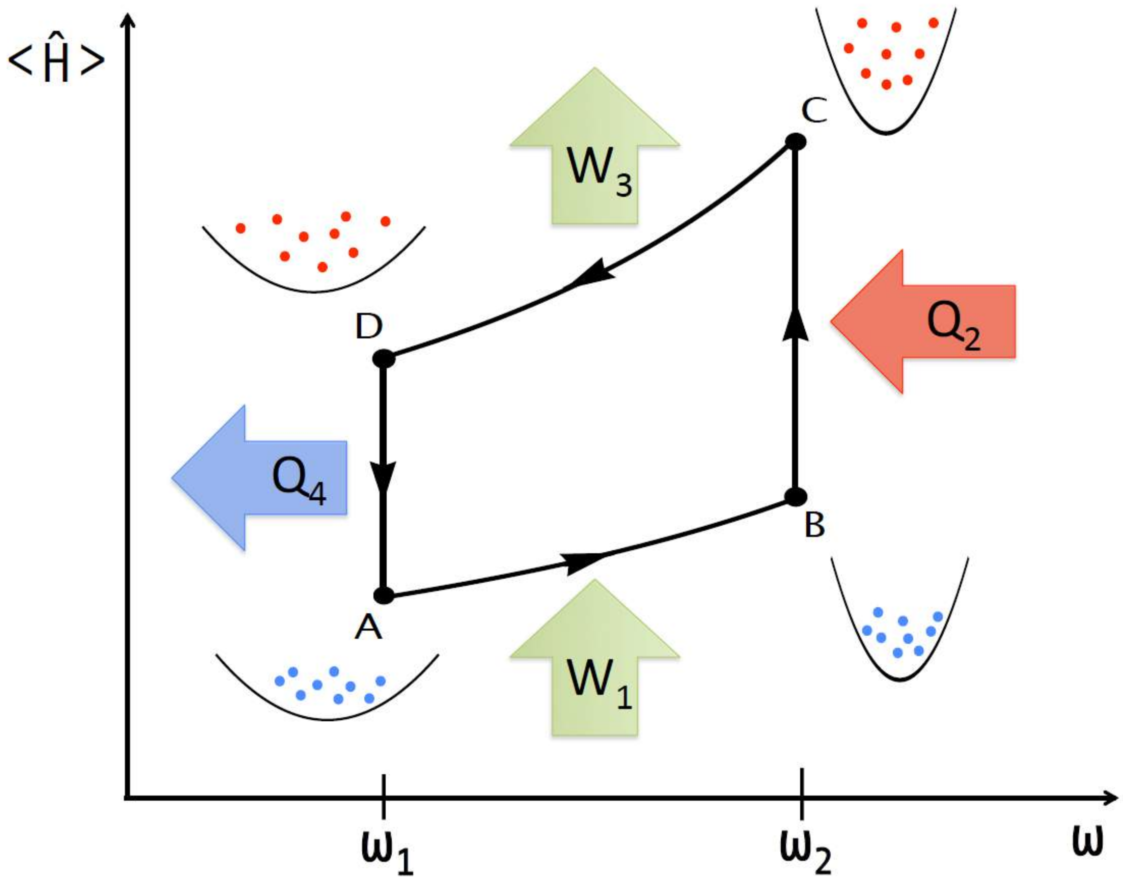
Many-particle QHE (Otto cycle)



Example: interacting Bose gas as working medium

Working medium - interacting many-particles with tunable interactions

$$H=\sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2}+\frac{1}{2}m\omega(t)^2x_i^2\right]+\frac{\hbar^2}{m}\sum_{i<j=1}^{N}\frac{\lambda(\lambda-1)}{(x_i-x_j)^2}$$

[1971: Calogero, J. Math. Phys. 12, 419; Sutherland, ibid 12, 246]

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Example: interacting Bose gas as working medium

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[1971: Calogero, J. Math. Phys. 12, 419; Sutherland, ibid 12, 246]

- Includes ideal bosons and hard-core bosons (= fermions) for λ=0,1
- Exact finite-time quantum thermodynamics – no approximations
- Equivalent to ideal gas of particles obeying fractional exclusion [Murthy & Shankar PRL 73, 3331 (1994)]
- Universal behavior (Luttinger liquid) of 1D many-body systems
- Tunable zero-point energy + linear spectrum

$$E(\{n_k\})=\frac{\hbar\omega}{2}N[1+\lambda(N-1)]+\sum_{k=1}^{\infty}\hbar\omega k n_k$$

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Quest for Quantum Supremacy

Comparison

Worst case: sudden-quench limit (sq)

- Efficiency ratio at maximum power

$$\rho_{\rm sq}^{(N)} := \frac{\eta_{\rm sq}^{(N)}}{\eta_{\rm sq}^{(1)}}$$

- Power ratio

$$r_{\rm sq}^{(N)} := \frac{P_{\rm sq}^{(N)}}{N P_{\rm sq}^{(1)}}$$

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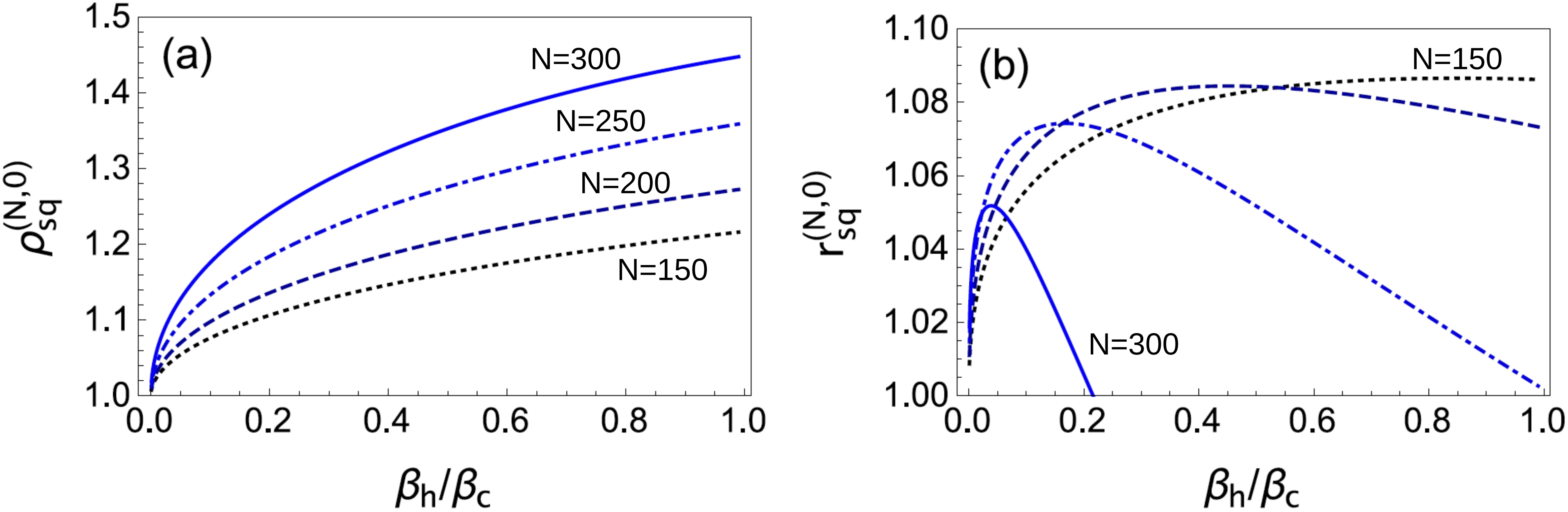
Quantum Supremacy: noninteracting case

Simultaneous enhancement of efficiency and power



Up to 50% efficiency enhancement

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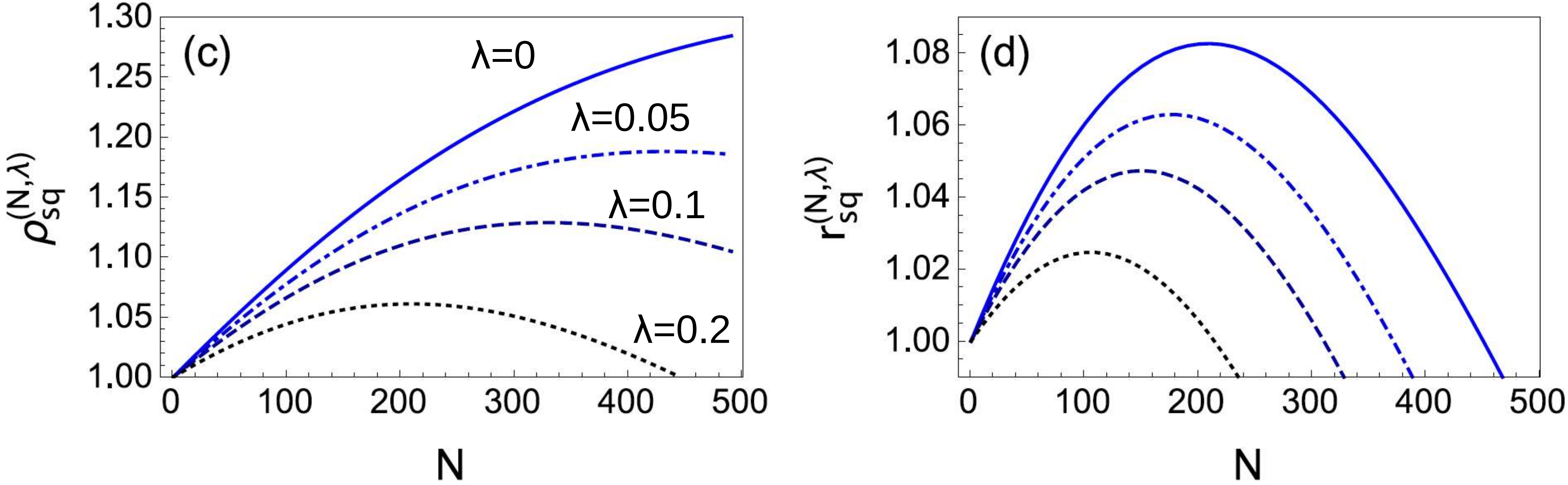
Quantum Supremacy: interacting case



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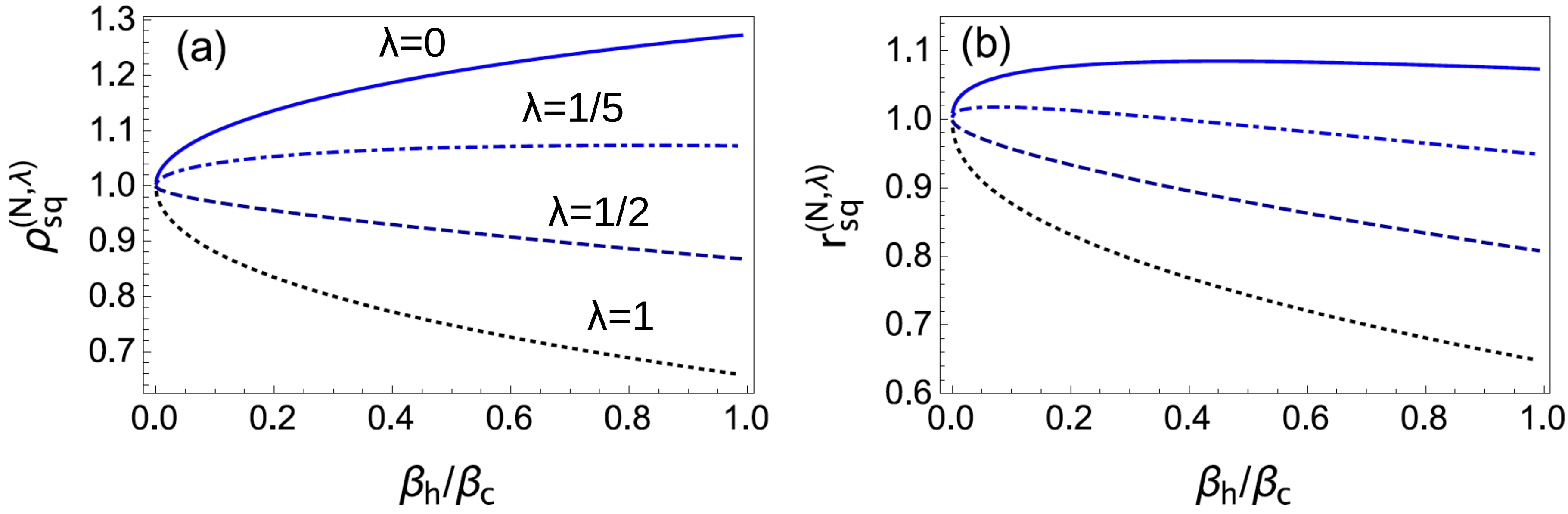
Quantum Supremacy

Simultaneous enhancement of efficiency and power (N=200)

Caveat: QS suppressed by strong interactions



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Summary

Shortcuts to adiabaticity speed up processes by tailoring excitations

- Three techniques:
  (1) inverting scaling laws,
  (2) counterdiabatic driving
  (3) fast-forward

- Applications
  Superadiabatic expansions/compressions
  Experimental test of counterdiabatic driving: continuous variables
  Supremacy of counterdiabatic transport
  STA in Quantum Thermodynamics

The Group

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Guan-Can Guo (Hefei)

Thanks for your attention!!

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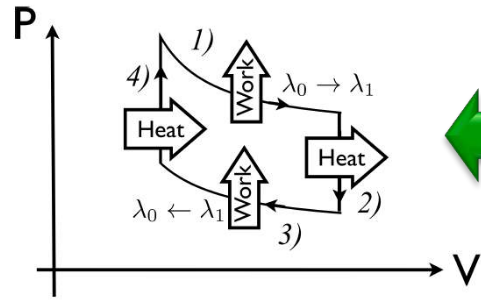
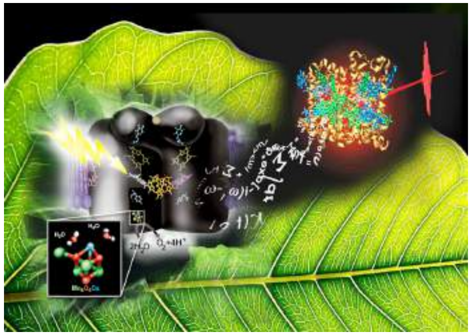
Quantum Heat Engines: Towards Green Quantum Energy

Optimal energy consumption and conversion

Equivalence Quantum engines & Photocells





Photosynthetic reaction center as a quantum heat engine

Konstantin E. Dorfman[a,b,c,1], Dmitri V. Voronine[a,b,1], Shaul Mukamel[c], and Marlan O. Scully[a,b,d]

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PNAS

PRL 111, 253601 (2013) PHYSICAL REVIEW LETTERS week ending 20 DECEMBER 2013

Efficient Biologically Inspired Photocell Enhanced by Delocalized Quantum States

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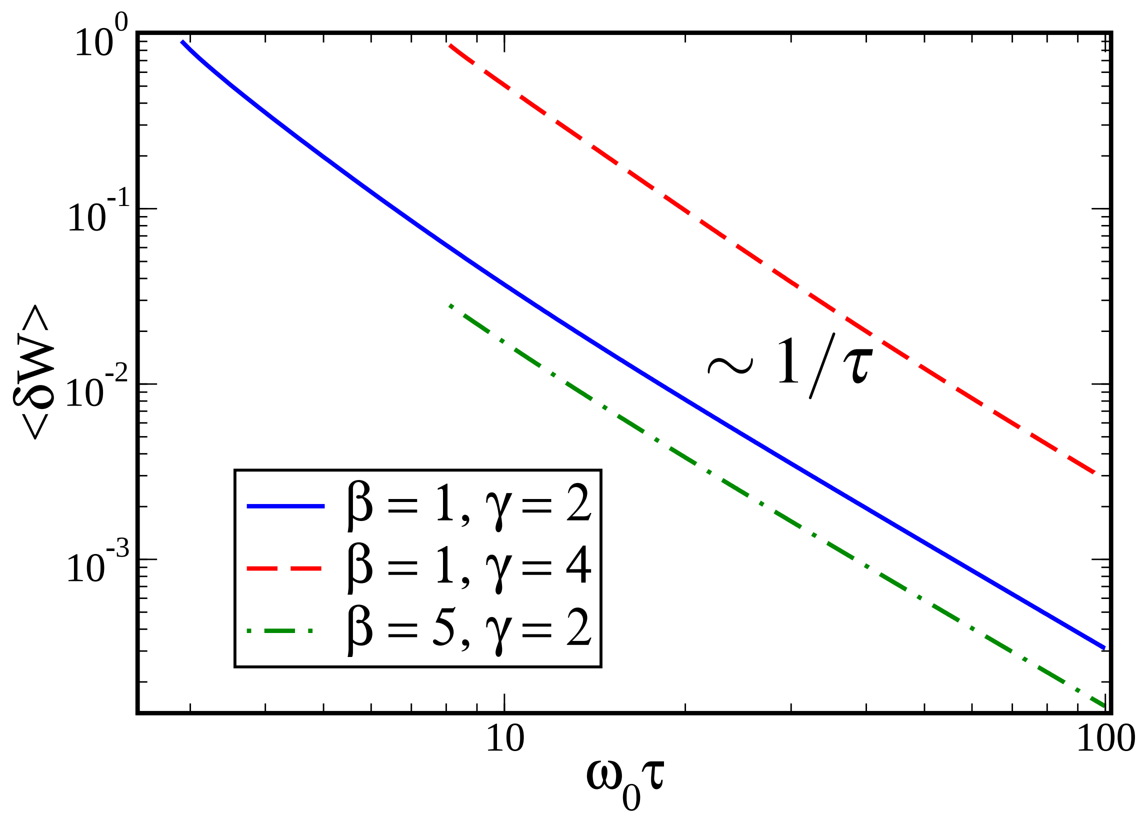
Energy Cost of Shortcuts to Adiabaticity

$$\delta W = \langle W \rangle - \langle W_{\rm ad} \rangle \qquad \langle \delta W \rangle = \frac{1}{\tau}\int_0^\tau \delta W dt$$



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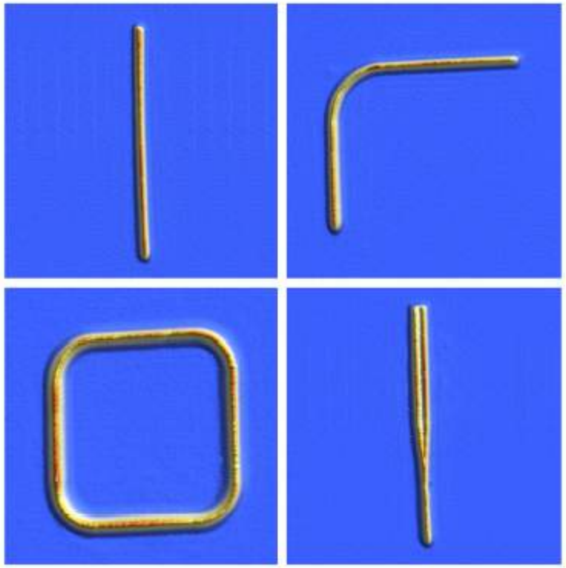
Part III

Design of bent waveguides
Tailoring curvature effects



del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)
Ryu & Boshier New J. Phys **17**, 092002 (2015)

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Curvature-induced potential (CIP)

- Waveguide with non-zero curvature
- Dimensional reduction of the Schrödinger equation under tight transverse confinement
- Emergence of quantum-mechanical local attractive potential

$$V_{\mathrm{CIP}}(q) = -\frac{\hbar^2}{8m}\kappa(q)^2$$

Curvature: rate of change of unit tangent vector $\kappa(q) = \left\| \frac{d\mathbf{T}}{dq} \right\|$

Switkes, Russel & Skinner, J. Chem. Phys. **67**, 3061(1977)
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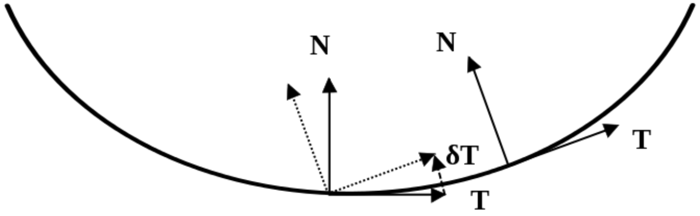
Exner & Seba, J. Math. Phys. **30**, 2574 (1989)



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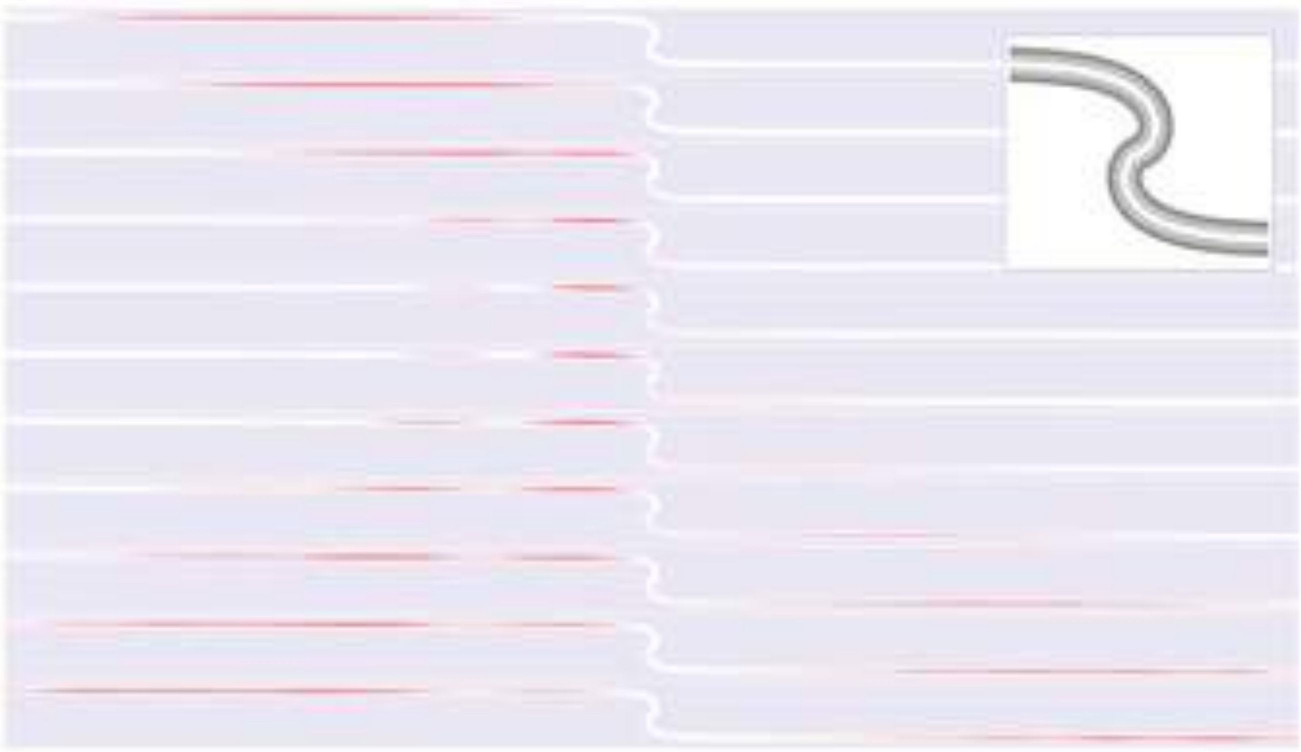
Curvature effects in atomtronics

Curvature affects scattering properties in atom circuits

Example: wavepacket splitting

time



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Supersymmetric reflectionless waveguides

- Supersymmetric quantum mechanics identifies families of reflectionless potentials

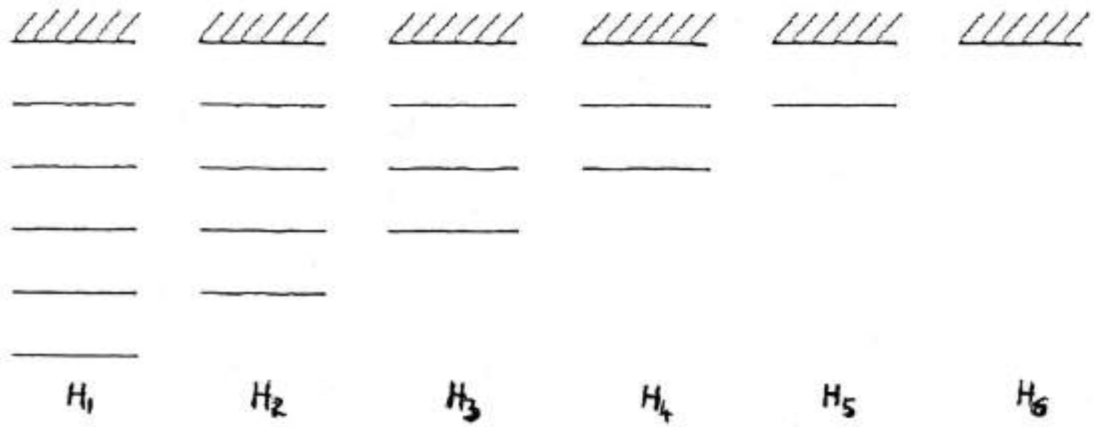

FIGURE 2. Schematic diagram of the eigenvalue spectra of the Hamiltonians in the hierarchy H_n. The number of bound states of H_1 is arbitrarily chosen to be 5.

SUSY partner Hamiltonians share scattering properties

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Supersymmetric reflectionless waveguides

- Supersymmetric quantum mechanics identifies families of reflectionless potentials

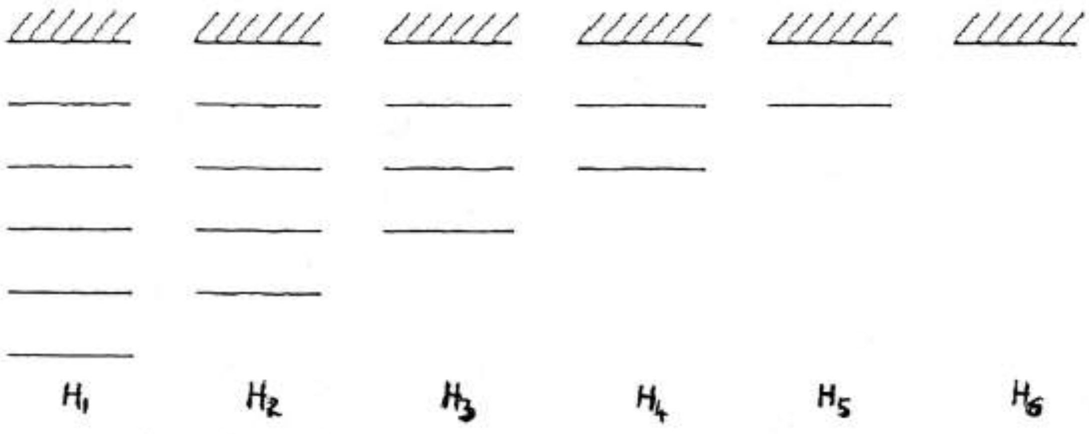

FIGURE 2. Schematic diagram of the eigenvalue spectra of the Hamiltonians in the hierarchy H_n. The number of bound states of H_1 is arbitrarily chosen to be 5.

SUSY partner Hamiltonians share scattering properties

Idea:
Design waveguides with a curvature-induced potential that is SUSY partners of V=0 (free dynamics/straight waveguide)
Reflectionless bent waveguides with unit transmission probability

del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

Supersymmetric reflectionless waveguides

- Supersymmetric quantum mechanics identifies families of reflectionless potentials

Unit transmission probability at any energy

- Curvature relation between SUSY waveguides

$$\kappa_+^2(q) = \kappa_-^2(q) + 8\left[\frac{\partial_q^2\psi_0}{\psi_0} - \left(\frac{\partial_q\psi_0}{\psi_0}\right)^2\right].$$

- Curvature specifies uniquely the waveguide shape (Frenet-Serret equations)

- Choose curvature to make CIP reflectionless, isospectral to straight waveguide

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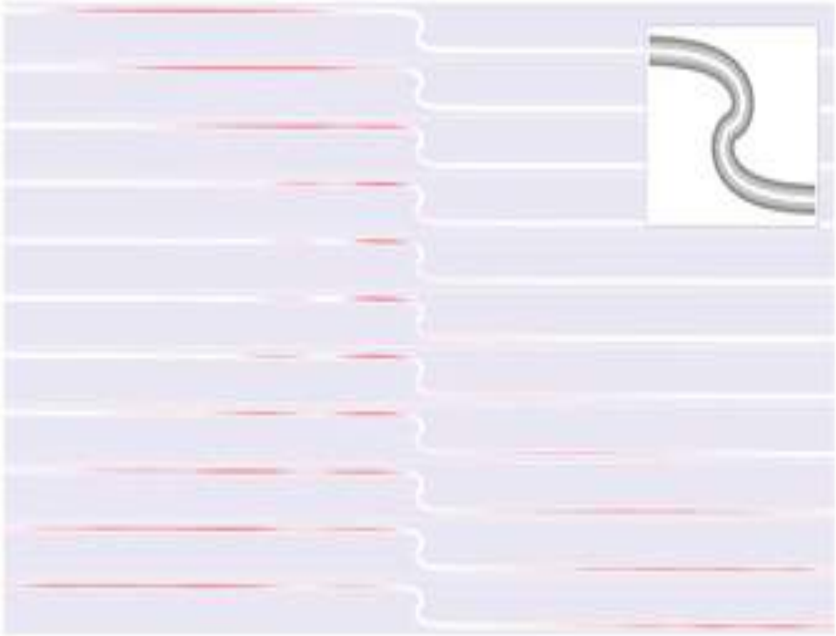
del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

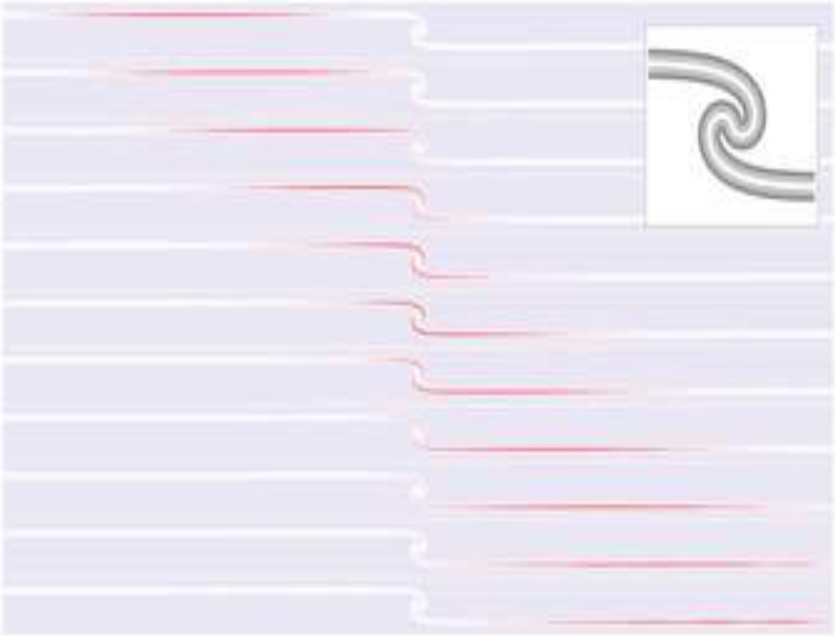
Supersymmetric reflectionless waveguides

- Supersymmetric quantum mechanics identifies families of reflectionless potentials
- Choose curvature to make CIP reflectionless

Curved waveguide



Curved SUSY waveguide



isospectral to straight waveguide

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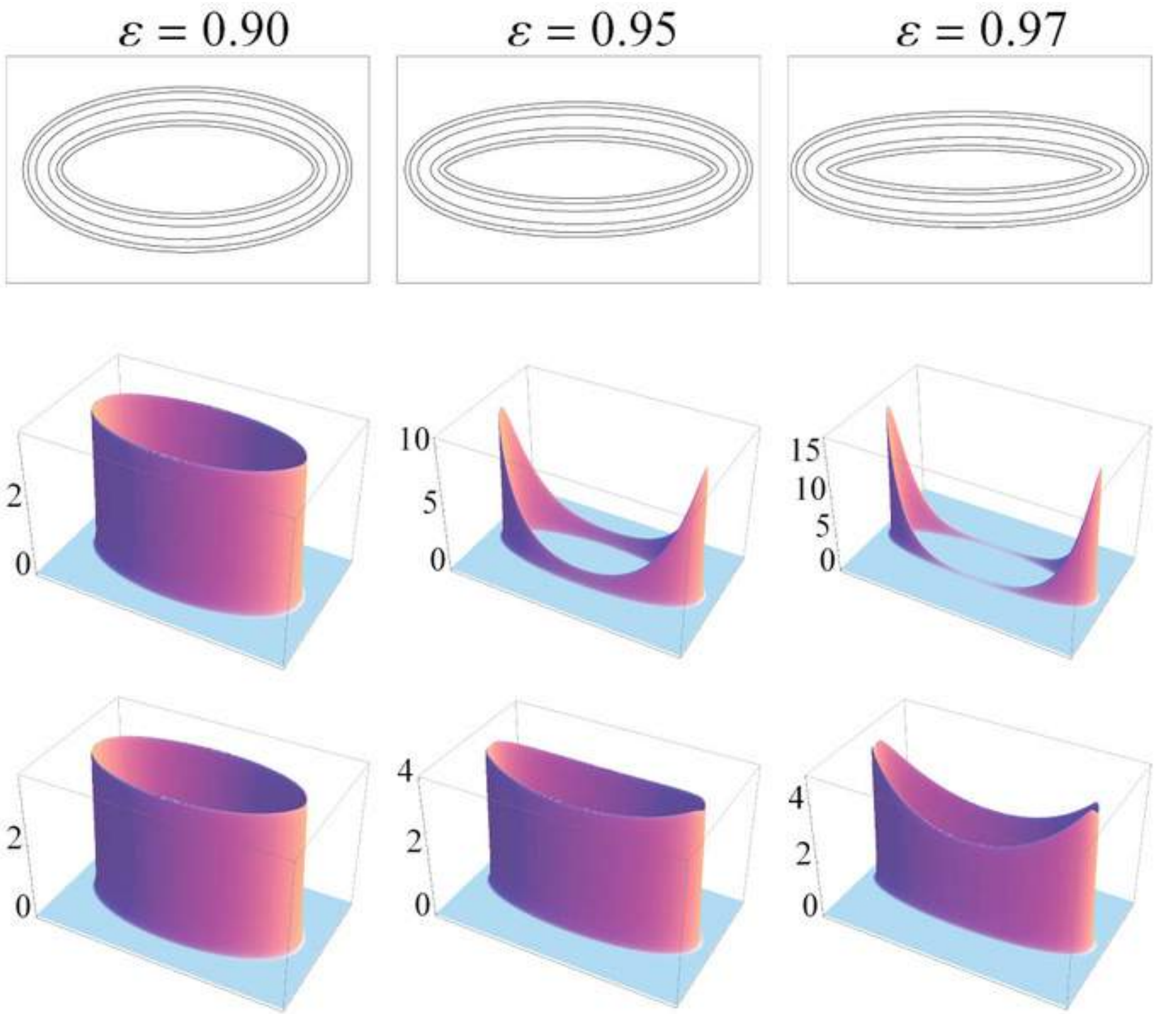
Curvature-induced effects: Elliptical waveguide potentials



del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)

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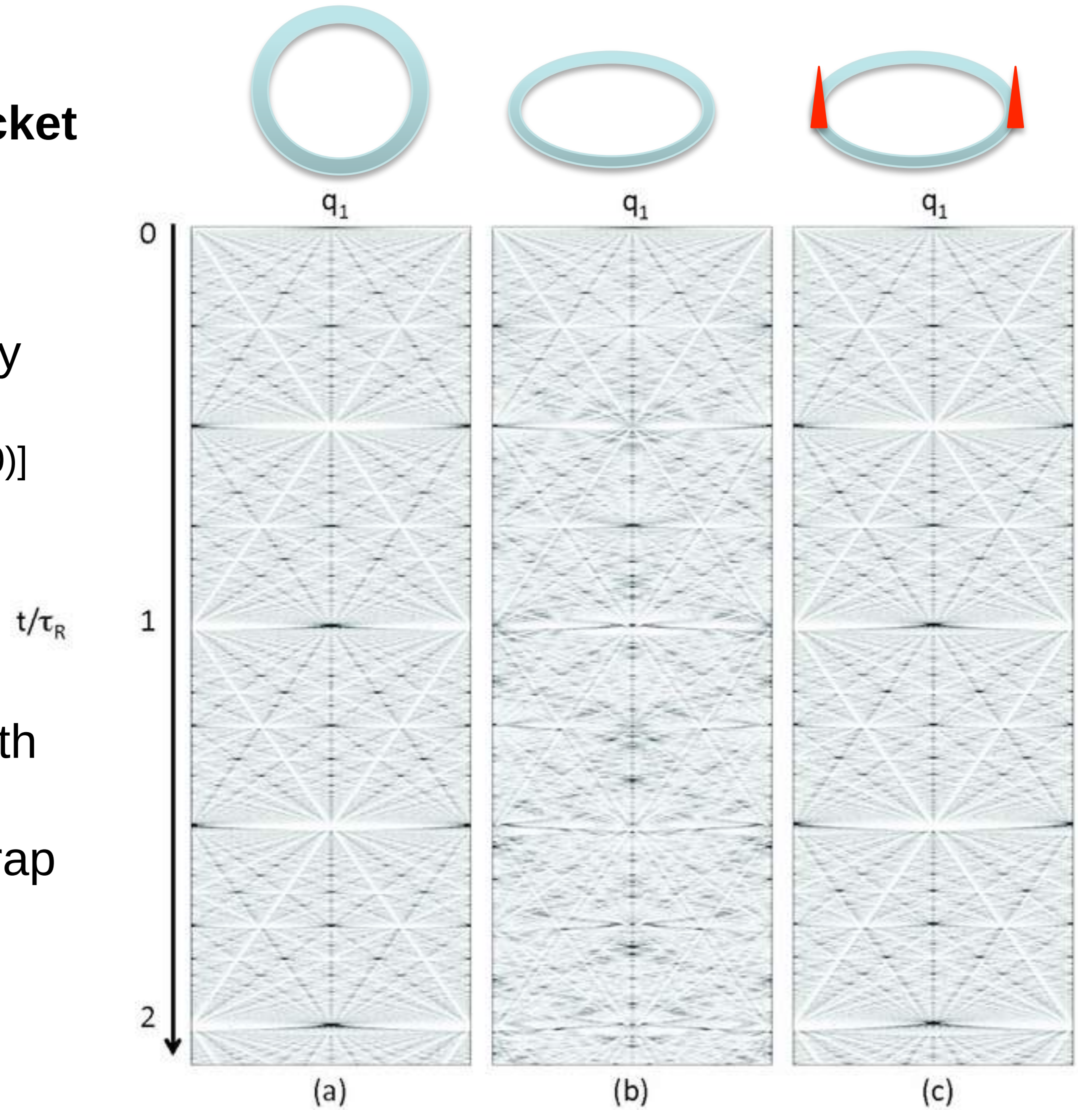
Quantum carpets: Elliptical waveguide potentials

Released localized wavepacket
Talbot oscillations
in the density profile

a) Periodic pattern in the density profile in a ring trap
[see Friesch et al. New J. Phys. **2**, 4 (2000)]

a) Suppressed by curvature in elliptical trap

b) Recovered in elliptical trap with cancelled curvature-induced potential: isospectral to ring trap



del Campo, Boshier, Saxena, Sci. Rep. **4**, 5274 (2014)